SURFACE AREAS AND VOLUMES

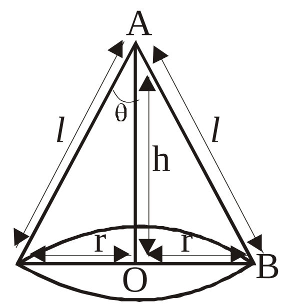
CONE

RIGHT CIRCULAR CONE :

The solid generated by the rotation of a right angled triangle about one of the sides containing the right angle is called a right circular cone.

Thus, on rotating a right angled triangular lamina AOB about OA, it generates a cone.

The point A is the vertex of the cone. Its base is a circle with centre O and radius OB.



The length OA is the height of the cone and the length AB is called its slant height. Clearly $\angle AOB = 90°$

If radius of base = r units

Height = h units and slant height = l units

Then $l^2 = (h^2 + r^2)$

at $l = \sqrt{h^2 + r^2}$

Formulae :

For a right circular cone of radius = r units, height = h units and slant height = l units, then

(i) Volume of the cone $= \frac{1}{3}\pi r^2 h$ cubic units

(ii) Curved surface area = $\pi r l$ sq units

(iii) Total surface area = curved surface area + area of the base

$= (\pi r l + \pi r^2)$ sq units $= \pi r (l + r)$ sq units

Ex.1 How many metres of cloth of 1.1 m width will be required to make a conical tent whose vertical height is 12 m and base radius is 16 m ? Find also the cost of the cloth used at the rate of Rs 14 per metre.

Sol. h = 12 m r = 16 m

$\therefore \quad \ell = \sqrt{r^2 + h^2}$

$= \sqrt{(16)^2 + (12)^2} = \sqrt{256 + 144}$

$= \sqrt{400} = 20\text{ m}$

$\therefore$ Curved surface area = $\pi r \ell = \frac{22}{7} \times 16 \times 20 = \frac{7040}{7}\text{ m}^2$

Width of cloth = 1.1 m

$\therefore$ Length of cloth = $\frac{7040/7}{1.1} = \frac{70400}{77} = \frac{6400}{7}\text{ m}$

$\therefore$ Cost of the cloth used @ Rs 14 per metre = Rs $\frac{6400}{7} \times 14$ = Rs 12800 **Ans.**

Ex.2 The ratio of the volumes of the two cones is 4 : 5 and the ratio of the radii of their bases is 2 : 3. Find the ratio of their vertical heights.

Sol. Let the radii of bases, vertically heights and volumes of the two cones be r_1, h_1, v_1 and r_2, h_2, v_2 respectively. According to the question,

$\frac{v_1}{v_2} = \frac{4}{5}$ ...(i) $\frac{r_1}{r} = \frac{2}{3}$ ....(i)

From (i), we have $\frac{\frac{1}{2}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{4}{5}$

$\Rightarrow \quad \frac{r_1^2 h_1}{r_2^2 h_2} = \frac{4}{5}$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 \frac{h_1}{h_2} = \frac{4}{5}$$

$$\Rightarrow \left(\frac{2}{3}\right)^2 = \frac{h_1}{h_2} = \frac{4}{5}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{4}{5}\left(\frac{3}{2}\right)^2 \qquad \text{[Using (ii)]}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{9}{5}$$

Hence the ratio of their vertical height is 9 : 5. **Ans.**

Ex.3 If h, c and v be the height, curved surface and volume of a cone, show that $3\pi v h^3 - c^2h^2 + 9v^2 = 0$.

Sol. Let the radius of the base and slant height of the cone be r and ℓ respectively.

The n ;

c = curved surface $= \pi r\ell = \pi r\sqrt{r^2 + h^2}$ ....(i)

v = volume $= \frac{1}{3}\pi r^2 h$ ...(ii)

$$\therefore \quad 3\pi v h^3 - c^2h^2 + 9v^2 = 3\pi\left(\frac{1}{3}\pi r^2 h\right)h^2 - \pi^2 r^2 (r^2 + h^2)h^2 + 9\left(\frac{1}{3}\pi r^2 h\right)^2 \qquad \text{[Using (i) and (ii)]}$$

$$= ^2 r^2h^4 - \pi^2r^2h^2 - \pi^2r^2h^4 + \pi^2r^4h^2 = 0.$$ **Hence Proved.**

Ex.4 By melting a solid cylindrical metal, a few conical materials are to be made. If three times the radius of the cone is equal to twice the radius of the cylinder an the ratio of the height of the cylinder and the height of the cone is 4 : 3, find the number of cones which can be made.

Sol. Let R be the radius and H be the height of the cylinder and let r and h be the radius and height of the cone respectively. Then,

3r = 2R

And H ; h = 4 : 3 ....(i)

$\Rightarrow \frac{H}{h}=\frac{4}{3}$

$\Rightarrow$ 3H = 4h ....(ii)

Let n be the required number of cones which can be made from the materials of the cylinder. Then, the volume of the cylinder will be equal to the sum of the volumes of n cones. Hence, we have

$$\pi R^2 H=\frac{n}{3}\pi r^2 h$$

$\Rightarrow 3R^2H = nr^2h$

$$\Rightarrow n=\frac{3R^2H}{r^2h}=\frac{3\times\frac{9r^2}{4}\times\frac{4h}{3}}{r^2h}$$ [$\because$ From (i) and (ii), $R=\frac{3r}{2}$ and $H=\frac{4h}{3}$]

$$=\frac{3\times9\times4}{3\times4}=9$$

Hence, the required number of cones is 9. **Ans.**