HERON'S FORMULA

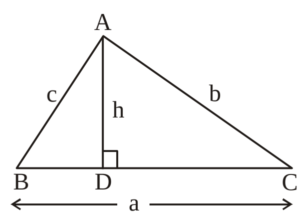
AREA OF TRIANGLE BY HERON'S FORMULA

INTRODUCTION

In earlier classes, we have learnt about various plane figure such as triangles, quadrilaterals, squares, rectangles etc. We have also learnt the formulae for finding the perimeter and area of square, rectangle, and some specific triangles. In this chapter we shall learn to find the area of a triangle in terms of the length of its "three sides" by using Heron's formula and application of this formula in finding areas of other rectilinear figures.

AREA OF TRIANGLE

If A, B and C are three non-collinear points, then the figure formed by the line segments AB, BC and CA is Δ ABC. These line segments are the sides and A, B and C are the vertices of the triangle. The sides BC, CA and AB are denoted respectively by a, b and c.



A triangle and its interior together are known as a triangular region.

- By the term area of a triangle we always mean the area of triangular region.
- The side on which a triangle stands is known as its base.
- The length of perpendicular from any vertex on the opposite side of a triangle is known as altitude or height of the triangle.

If A denotes the area of a triangle whose base is b and height is h then,

$$A = \frac{1}{2} bh$$

Also, $h = \frac{2A}{b}$ (When area and base are given)

and $b = \frac{2A}{h}$ (When area and height are given)

1 The Heron's Formula (Hero's Formula)

We have calcualted the areas of triangle when base and height are given (or can be calculated as in case of isosceles and equilateral triangles). However, when the triangle is scalene and its three sides are given, it is not possible to find the height by the methods studied till now, we can not apply the usual formula i.e., ½ base × height

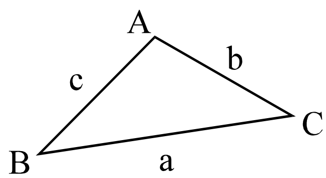
The famous first century Greek mathematician Heron of Alexandria invented a formula to find the area of a triangle whose three sides are known. His three books on mensuration were so popular that mathematicians called him Hero of Alexandria.

- Heron's formula for finding area of a triangle in terms of its sides is stated as.

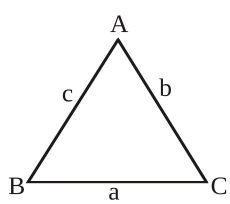
  Area of a triangle $=\sqrt{s(s-a)(s-b)(s-c)}$

  where a, b and c are the sides of the triangle and

  $s = \frac{a+b+c}{2}$, is the semi perimeter (half of the perimeter) of the triangle.



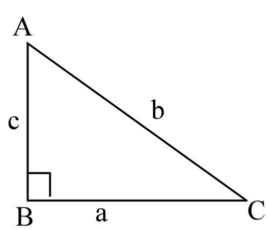
- Area of an equilateral triangle = $\frac{\sqrt{3}}{4}(\text{side})^2$
- Area of an isosceles triangle $=\frac{1}{4}a\sqrt{4b^2-a^2}$ (where b is length of equal sides)



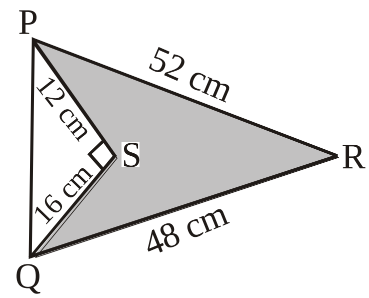
Pythagoras Theorem

In a right angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$AC^2 = AB^2 + BC^2$



Ex.1 Find the area of the shaded portion of the Δ as shown in figure.



Sol. In right ΔPSQ, by Pythagoras Theorem

$PQ^2 = PS^2 + SQ^2$

$= (12)^2 + (16)^2 = 144 + 256 = 400 \text{ cm}^2$

PQ = cm = 20 cm.

Now a = 20 cm, b = 48 cm and c = 52 cm.

$\therefore s = \frac{a+b+c}{2} = \frac{20cm + 48cm + 52cm}{2} = \frac{120cm}{2} = 60cm$

Area of $\Delta PQR = \sqrt{s(s-a)(s-b)(s-c)}$

$\sqrt{60\,(60-20)\,(60-48)\,(60-52)}$

$= \sqrt{60\times 40\times 12\times 8} = \sqrt{6\times 10\times 10\times 4\times 6\times 2\times 8}$

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$= 6 \times 10 \times 8 = 480 \text{ cm}^2$

and area of ΔPQS

$\frac{1}{2}\times PS\times QS = \frac{1}{2}\times 12\times 16 = 96cm^2$

Area of the shaded portion of the Δ = 480 – 96 = 384 cm^2

Ex.2 The perimeter of an isosceles triangle is 42 cm and its base is $\frac{3}{2}$ times each of the equal sides. Find the length of each side of the triangle, area of the triangle and height of the triangle.

Sol. Let the equal sides be a and unequal side be b.

$2a + b = 42$

Given : $b = \frac{3}{2}a$

$2a + \frac{3}{2}a = 42$

$\frac{7}{2}a = 42$

a = 12 cm.

So, $b = \frac{3}{2}(12) = 18$ cm.

Also, perimeter = 2s = 42 cm

$s = 21$ cm.

Area of triangle $= \sqrt{21(21-12)(21-12)(21-18)}$

$= \sqrt{21(9)(9)(3)} = 27\text{cm}^2$

Area of triangle $= \frac{1}{2} \times$ base $\times$ height

$27\sqrt{7} = \frac{1}{2} \times 18 \times$ height

Height $= \frac{54\sqrt{7}}{18} = 3\sqrt{7}$ cm.

Ex.3 Find the percentage increase in the area of a triangle if its each side is doubled.

Sol. Let a,b,c be the sides of the given triangle and s be its semi-perimeter

$\therefore \quad s = \frac{1}{2}(a+b+c)$ ....(i)

The sides of the new triangle are 2a, 2b and 2c.

Let s' be its semi-perimeter.

$\therefore \quad s' = \frac{1}{2}(2a+2b+2c) = a+b+c = 2s$ [Using (i)]

Let Δ = Area of given triangle

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ .....(ii)

And, Δ' = Area of new triangle

$\Delta' = \sqrt{s'(s'-2a)(s'-2b)(s'-2c)}$

$= \sqrt{2s(2s-2)(2s-2b)(2s-2c)}$ [Using (i)]

$= \sqrt{16s(s-a)(s-b)(s-c)}$

$\Delta' = 4\Delta$

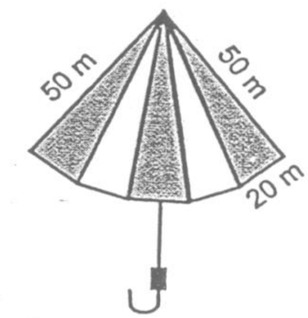
$\therefore$ Increase in the area of the triangle $= \Delta' - \Delta = 4\Delta - \Delta = 3\Delta$

$\therefore$ % increase in area $= \left(\frac{3\Delta}{\Delta} \times 100\right)\% = 300\%$ **Ans.**

Ex.4 An umbrella is made by stitching 10 triangular pieces of cloth of two different colors (see figure), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each color is required for the umbrella ?

Sol. The sides of a triangular piece are

20 cm, 50 cm and 50 cm



$s = \frac{a+b+c}{2} = \frac{20+50+50}{2} = 60\text{cm}$ = 60 cm

Area of one triangular piece

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-20)(60-50)(60-50)}$

$= \sqrt{60 \times 40 \times 10 \times 10} = \sqrt{2400}$

$= 200\sqrt{6}\ \text{cm}^2$

Area of cloth of each colour for five triangular pieces

$= 5 \times 200\sqrt{6} = 1000\sqrt{6}\ \text{cm}^2$ **Ans.**

Ex.5 Find the area of a triangle whose sides are of lengths 52 cm, 56 cm and 60 cm respectively.

Sol. Let a = 52 cm, b = 56 cm and c = 60 cm.

Perimeter of the triangle = (a + b + c) units

= (52 + 56 + 60) cm = 168 cm

$\therefore \quad s = \frac{1}{2}(a+b+c) = \left(\frac{1}{2} \times 168\right)\text{cm}$ = 84 cm

(s – a) = (84 – 52) cm = 32 cm,

(s – b) = (84 – 56) cm = 28 cm

and (s – c) = (84 – 60) cm = 24 cm

By Heron’s formula, the area of the given triangle is

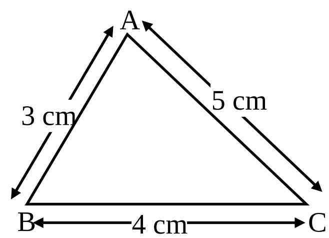
$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{84 \times 32 \times 28 \times 24}\ \text{cm}^2$$

$$= \sqrt{14 \times 6 \times 16 \times 2 \times 14 \times 2 \times 6 \times 4}\ \text{cm}^2$$

$$= (14 \times 6 \times 4 \times 2 \times 2)\ \text{cm}^2 = 1344\ \text{cm}^2$$

Ex.6 For given figure find the s (s – a).



Sol. Perimeter = 2s = 3 + 4 + 5 = 12 cm

$\therefore$ semi perimeter = $\frac{12}{2}$ = 6 cm

$\therefore$ s (s – a) = 6 (6 – 4)

= 6 × 2

= 12 cm.

Ex.7 If semi perimeter of a triangle is 60 cm & its two sides are 45 cm, 40 cm then find third side.

Sol. Semi perimeter = 60

$\therefore$ Perimeter = 2 × 60

$\Rightarrow$ Sum of all three sides = 120 (Let third side = x cm)

$\Rightarrow$ x + 45 + 40 = 120

$\Rightarrow$ x + 85 = 120

$\Rightarrow$ x = 120 – 85

$\Rightarrow$ x = 35 cm.