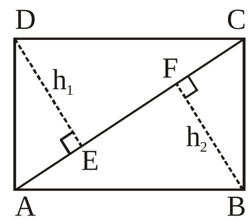
HERON'S FORMULA

APPLICATION OF HERON'S FORMULA IN FINDING

AREA OF QUADRILATERAL

MENSURATION OF A QUADRILATERAL :

1. When a diagonal and perpendiculars to the diagonals are given :



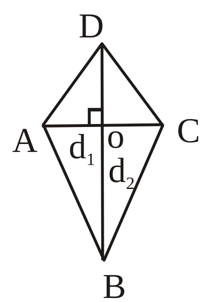
Given: AC = d

Perpendicular DE = h_1 and BF = h_2.

Area of quadrilateral ABCD = Area (ΔADC) + Area (ΔABC)

Area (ABCD) $= \frac{1}{2} AC \times h_1 + \frac{1}{2} AC \times h_2 = \frac{1}{2} AC(h_1 + h_2)$

2. When the diagonals are mutually perpendicular :



Given AC = d_1, BD = d_2.

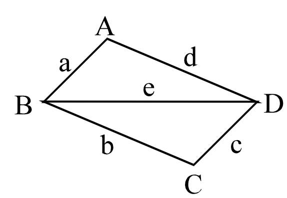
BD is the perpendicular bisector of AC.

Area (ABCD) = Area of ΔABD + Area of ΔCBD

$= \frac{1}{2} d_2 \times \frac{d_1}{2} + \frac{1}{2} d_2 \times \frac{d_1}{2} = \frac{1}{2}\left(2 \times \frac{d_1 d_2}{2}\right)$

$= \frac{1}{2} d_1 d_2 = \frac{1}{2}$ (product of the diagonals)

3. When the four sides and a diagonal are given



Given sides measuring a, b, c, d and diagonal measuring e.

The area of the quardrilateral (ABCD)

= area of ΔABD + area of ΔBCD

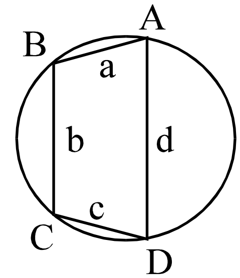
(area of ΔABD and ΔBCD can be obtained by using Hero's formula)

4. Area of a cyclic quadrilateral

Cyclic quadrilateral or Inscribed quadrilateral :

In Euclidean geometry a cyclic quadrilateral or inscribed quadrilateral is the quadrilateral whose vertices all lie on a single circle. This circle is called circumcircle or circumscribed circle and the vertices are said to be concyclic. The center of the circle and radius are called the circumcenter and the circumradius respectively. Other names for these quadrilaterals are concyclic quadrilaterals and chordal quadrilaterals.

Given cyclic quadrilateral ABCD with sides measuring a, b, c, d.



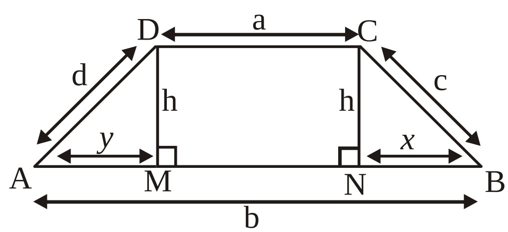
Area of cyclic quadrilateral = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$

(This formula is given by Brahmagupta. So this formula is also known as Brahmgupta's formula)

where $\mathbf{s} = \frac{a+b+c+d}{2}$

1 Mensuration of Trapezium

ABCD is a trapezium with AB || CD, AB = b, BC = c, CD = a, DA = d and the height of the trapezium ABCD is h, then



(1) Perimeter = a + b + c + d

(2) Area (ABCD) = Area of ΔAMD + Area of rectangle MNCD + Area of ΔCNB

$$= \frac{1}{2}.y.h + a.h. + \frac{1}{2}.x.h = \frac{1}{2}h(x+y) + ah$$

$$= \frac{1}{2}h[x+y+2a] = \frac{1}{2}h[(x+y+a)+a]$$

$$= \frac{1}{2}h[b+a] = \frac{1}{2}(a+b)h$$

$$= \frac{1}{2}(\text{sum of the parallel sides}) \times (\text{distance between them})$$

2 Mensuration of a Parallelogram

1. Perimeter of a parallelogram given its sides

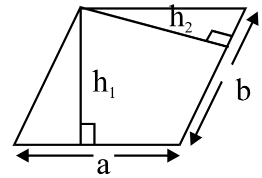
   Perimeter = 2 (sum of the adjacent sides) = 2(a + b)

2. Area of a parallelogram when base and corresponding altitude is given

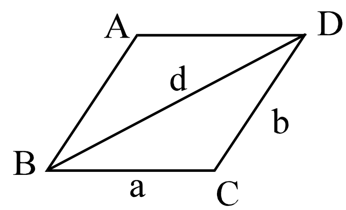
(a) Area = $ah_1 = bh_2$ (b) $a = \frac{Area}{h_1}$

(c) $b = \frac{Area}{h_2}$ (d) $h_1 = \frac{Area}{a}$

(e) $h_2 = \frac{Area}{b}$



3. Area when a diagonal and two sides given



Given sides a, b and Length of one diagonal = d.

Area of Δ BCD = $\sqrt{s(s-a)(s-b)(s-d)}$

where $s = \frac{a+b+d}{2}$

Also, area of Δ ABD = area of Δ BCD [Diagonal bisects a parallelogram]

area of the parallelogram = $2\sqrt{s(s-a)(s-b)(s-d)}$

where $s = \frac{a+b+d}{2}$

3 Mensuration of a Rhombus

1. When two diagonals are given

Length of diagonals d_1, d_2

(a) Area = $\frac{1}{2} d_1 d_2$

(b) Side $= \sqrt{\frac{d_1^2}{4} + \frac{d_2^2}{4}} = \frac{1}{2}\sqrt{d_1^2 + d_2^2}$

(c) Perimeter = $2\sqrt{d_1^2 + d_2^2}$

2. When side and one diagonal are given

Side = a

One diagonal = d_1

d_2 = Other diagonal $= \sqrt{a^2 - \frac{d_1^2}{4}} = \frac{1}{2}\sqrt{4a^2 - d_1^2}$

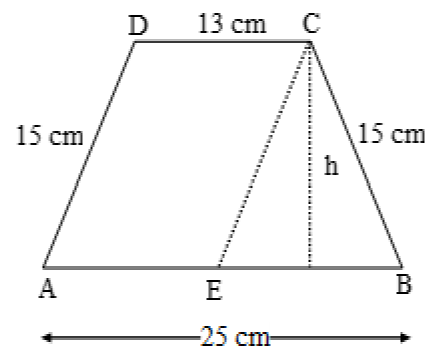
Area = $=\frac{1}{2}\times d_1\times\frac{1}{2}\sqrt{4a^2-d_1^2}=\frac{1}{4}\times d_1\sqrt{4a^2-d_1^2}$

Ex.1 Find the area of a trapezium whose parallel sides 25 cm, 13 cm and other sides are 15 cm and 15 cm.

Sol. Let ABCD be the given trapezium in which AB = 25 cm, CD = 13 cm, BC = 15 cm and AD = 15 cm.

Draw CE || AD.

Now, ADCE is a parallelogram in which AD || CE and AE || CD.



$\therefore$ AE = DC = 13 cm and BE = AB – AE

= 25 – 13 = 12 cm

In Δ BCE, we have

$s=\frac{15+15+12}{2}=21$

$\therefore$ Area of ΔBCE = $\sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow$ Area of ΔBCE

= $\sqrt{21(21-15)(21-15)(21-12)}$

$\Rightarrow$ Area of ΔBCE

= $\sqrt{21\times6\times6\times9}=18\sqrt{21}\text{cm}^2$ ....(i)

Let h be the height of ΔBCE, then

Area of ΔBCE = $\frac{1}{2}$ (Base × Height)

= $\frac{1}{2}$ × 12 × h = 6h .....(ii)

From (i) and (ii), we have,

6h = $8\sqrt{21}\Rightarrow$ h = $3\sqrt{21}$cm

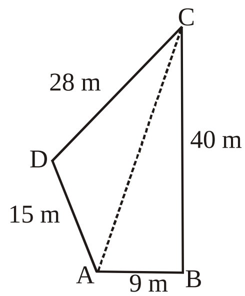
Clearly, the height of trapezium ABCD is same as that of ΔBCE.

$\therefore$ Area of trapezium = $\frac{1}{2}$ (AB + CD) × h

$\Rightarrow$ Area of trapezium

$= \frac{1}{2}(25 + 13) \times 3\sqrt{21}\ \text{cm}^2 = 57\sqrt{21}\ \text{cm}^2$

Ex.2 Find the area of a quadrilateral ABCD whose sides are 9 m, 40 m, 28 m and 15 m respectively and the angle between the first two sides is a right angle.



Sol. Let ABCD be the given quadrilateral such that $\angle ABC = 90^o$ and AB = 9 m, BC = 40 m, CD = 28 m, AD = 15 m.

In ΔABC, we have

$AC^2 = AB^2 + BC^2$ [Using Pythagoras Theorem]

$\Rightarrow AC^2 = 9^2 + 40^2 = 1681$

$\Rightarrow$ AC = 41 m

Now, Area of $\Delta ABC = \frac{1}{2}(Base \times Height)$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2}(AB \times BC)$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2}(9 \times 40)m^2 = 180 m^2$

In ΔACD, we have

AC = 41 m, CD = 28 m and DA = 15 m.

Let a = AC = 41 m, b = CD = 28 m and c = DA = 15m.

Then,

$s = \frac{1}{2}(a + b + c) = \frac{1}{2}(41 + 28 + 15) = 42$

$\therefore$ Area of ΔACD = $=\sqrt{(s-a)(s-b)(s-c)}$

$\Rightarrow$ Area of ΔACD $=\sqrt{42(42-41)(42-28)(42-15)}$

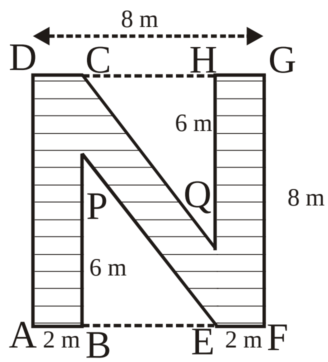
$\Rightarrow$ Area of ΔACD $=\sqrt{42\times1\times14\times27}=\sqrt{14\times3\times14\times27}$

$= 14 \times 9 = 126 \text{ cm}^2$

Area of quadrilateral ABCD = Area (ΔABC) + Area (ΔACD)

$= 180 \text{ m}^2 + 126 \text{ m}^2 = 306 \text{ m}^2$

Ex.3 In a square lawn of side 8 m, an N-shaped path is made, as shown in figure. Find the area of the path so shaded.



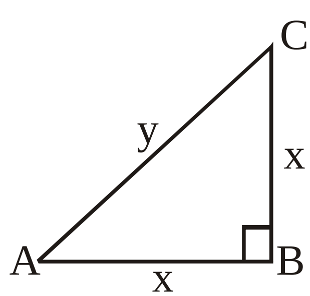
Sol Required area = ar. of shaded portion

= Ar. of sq. ADGF – ar. of ΔCQH – ar. of ΔPBE

$=\left[8\times8-\frac{1}{2}\{8-(2+2)\}\times6-\frac{1}{2}\{8-(2+2)\times6\}\right]\text{m}^2$

$= (64 - 12 - 12) \text{ m}^2 = 40 \text{ m}^2$

Ex.4 An isosceles right triangle has area 200 cm^2. What is the length of its hypotenuse?



Sol Let each equal side in right angled ΔABC be x cm

i.e. AB = BC = x cm

and its hypotenuse = AC = y cm

Area (ΔABC) $= 200 \text{ cm}^2$

$\frac{1}{2}.x.x = 200 \text{ cm}^2$

$x^2 = 400 \text{ cm}^2$

$x = \sqrt{400} = 20 \text{ cm}$

By Pythagoras Theorem, we have

$y^2 = x^2 + x^2$

$= (20)^2 + (20)^2 = 800$

$y = \sqrt{400 \times 2} = 20\sqrt{2} \text{ cm}$

Hence hypotenuse of right angled $\Delta = 20\sqrt{2}$ cm