CONSTRUCTIONS

BASIC CONSTRUCTION

INTRODUCTION

The diagrams, which were necessary to prove theorems or solving the problems were not necessarily very accurate. They were drawn only to give you a feeling for the situation and as an add for proper reasoning. But many times, we need to draw accurate figures. For examples to draw road map of a city, to draw layout plan of a building to draw a design of a vehicle etc. To draw such accurate figures, we must have some basic knowledge of geometrical constructions, which is a process of drawing a geometrical figure using some geometrical instruments like graduated ruler, protector, compass, a pair of set – squares.

In earlier classes, you have learned some simple geometrical constructions like drawing a line segment of a given measurement, making an angle with the help of protector etc. In this chapter; you will learn some more basic geometrical constructions using ruler and compass with reasoning behind it, why these constructions are valid.

TO CONSTRUCT THE BISECTOR OF A GIVEN ANGLE

Steps of Construction :

Step 1: Draw a line segment AB.

Step 2: With A as centre and any radius, draw an arc, cutting AB at P.

Step 3: With P as centre and the same radius as above, draw an arc cutting the previous arc at Q.

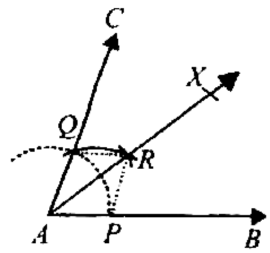
Step 4: Join AQ

Step 5: With P as centre and a radius more than $\frac{1}{2}$ PQ, draw an arc.

Step 6: With Q as centre and with the same radius, draw another arc, cutting the previous arc at R.

Step 7: Join AR.

Thus, AR is the required bisector of $\angle$BAC.



Justification :

Now let us see how this method gives us the required angle bisector. Join PR and QR.

In triangles APR and AQR, we have

AP = AQ [AP and AQ are radii of the same arc]

PR = QR [PR and QR are arcs of equal radii]

AR = AR [Common]

$\Delta APR \cong \Delta AQR$ [By SSS criteria]

$\Rightarrow \angle PAR = \angle QAR$ [C.P.C.T.]

Hence, AR is the bisector of $\angle BAC$.

TO CONSTRUCT THE PERPENDICULAR BISECTOR OF A GIVEN LINE SEGMENT

Given a line segment LM, we have to draw the perpendicular bisector.

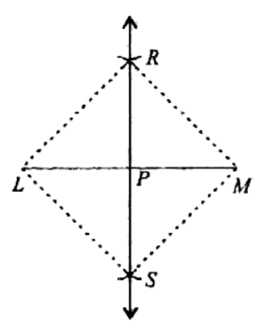
Steps of Construction :

Step 1: Draw a line segment $\frac{1}{2}$ LM of any given length.

Step 2: Taking L and M as centres and radius more than LM draw arcs on both sides of the line segment LM (tointersect each other)

Step 3: Let these arcs intersect each other at R and S.

Step 4: Join RS intersecting LM at P. Then P bisects the line segment LM as shown in figure. Join LR, RM, MS and LS.



Justification

In ΔRLS and ΔRMS,

$LR = MR$	[Arcs of equal radii]
$LS = MS$	[Arcs of equal radii]
$RS = RS$	[Common]
$\Delta RLS \cong \Delta RMS$	[By SSS criteria]
$\therefore \angle LRP = \angle MRP$	[C. P. C. T]

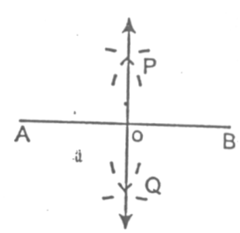
Now, in ΔRPL and ΔRPM

$LR = MR$	[Arcs of equal radii]
$\angle LRP = \angle MRP$	[Proved above]
$RP = RP$	[Common]
$\therefore \Delta RPL \cong \Delta RPM$	[By SAS criteria]
$LP = MP$ and $\angle RPL = \angle RPM$	[C. P. C. T]
$\angle RPL + \angle RPM = 180°$	[Linear pair axiom]

$\therefore \angle RPL = \angle RPM = 90°$

$\therefore$ RPS is the perpendicular bisector of LM.

Ex.1 Draw a line segment of length 7.8 cm draw the perpendicular bisector of this line segment.



Sol. Given the given the segment be $AB = 7.8$ cm.

STEPS :

(i) Draw the line segment $AB = 7.8$ cm.

(ii) With point A as centre and a suitable radius, more than half the length of AB, draw arcs on both the sides of AB.

(iii) With point B as centre and with the same radius draw arcs on both the sides of AB. Let these arc cut at points P & Q as shown on in the figure.

(iv) Draw a line through the points P and Q. The line so obtained is the required perpendicular bisector of given line segment AB.

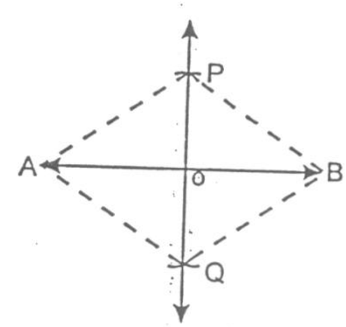
Line PQ is perpendicular bisector of AB.

(A) PQ bisects AB i.e., $OA = OB$.

(B) PQ is perpendicular to AB i.e., $\angle PAO = \angle POB = 90^0$.

Proof :

In ΔAPQ and ΔBPQ :



$AP = BP$ [By construction]

$AQ = BQ$ [By construction]

$PQ = PQ$ [Common]

$\Rightarrow$ $\Delta APQ = \angle BPQ$ [By SSS]

$\Rightarrow$ $\angle APQ = \angle BPQ$ [By cpctc]

Now, in Δ APO & ΔBPO

$AP = BP$ [By construction]

$OP = OP$ [Common side]

$\angle APO = \angle BPO$ [Proved above]

$\Rightarrow$ $\Delta APO \cong \Delta BPO$ [By SAS]

And, $\angle POA = \angle POB$

$= \frac{180^0}{2} = 90^0$ [$\because \angle POA + \angle POB = 180^0$]

$\Rightarrow$ PQ is perpendicular bisector of AB.

CONSTRUCT OF STANDARD ANGLES

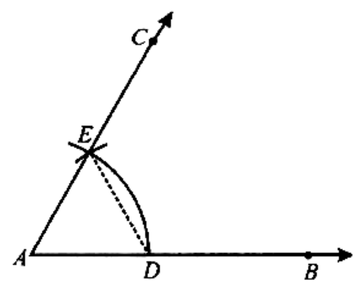
Given a ray AB, we want to construct another ray AC such that $\angle CAB = 60°$

Steps of Construction :

Step 1: Taking A as centre and any suitable radius, draw an arc to intersect the ray AB at D.

Step 2: Taking D as centre and same radius as in step (i), draw another arc to intersect the previous arc at E.

Step 3: Draw ray AC passing through E. Then $\angle CAB = 60°$.



Justification:

Join DE.

Then AD = AE = DE

$\therefore$ ΔADE is equilateral $\Rightarrow \angle EAD = 60°$

$\therefore$ $\angle CAB = 60°$

Ex.2: Construct an angle 120°.

Sol: **Steps of Construction**

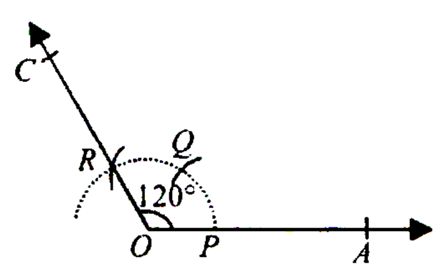
Step 1: Draw a ray OA.

Step 2: With O as centre and any radius, draw an arc cutting OA at P.

Step 3: With P as centre and the same radius draw an arc, cutting the first arc at Q.

Step 4: With Q as centre and the same radius, draw an arc, cutting the arc drawn in step 2 at R.

Step 5: Join OR and produce it to any point C. $\angle AOC$ so obtained is the angle of measure 120°.



To Construct and Angle of 30^0 :

STEPS :

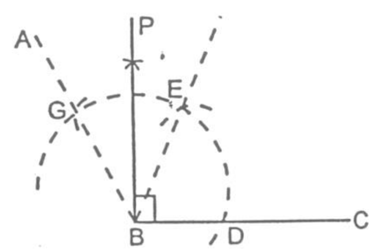
(i) Construct angle ABC $= 60^0$ by compass.

(ii) Draw BD, the bisector of angle ABC.

The, $\angle DBC = 30^0$

To Construct an Angle of 90^0 :

STEPS

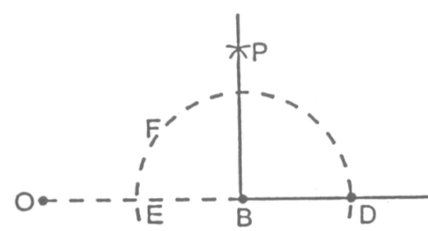


(i) Construct angle ABC = 120.0 by using compass.

(ii) Draw PB, the bisector of angle EBG.

Then, $\angle PBC = 90^0$

Alternative Method :



(i) Draw a line segment BC of any suitable length.

(ii) Produce CB upto a arbitrary point O.

(iii) Taking B as centre, draw as arc which cuts OC at points D and E.

(iv) Taking D and E as centres and with equal radii draw arcs with cut each other at point P.

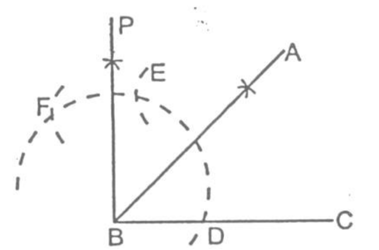
[The radii in this step must be of length more than half of DE.]

(v) Join BP and produce.

Then, $\angle PBC = 90^0$

To Construct an Angle of 45^0

STEPS



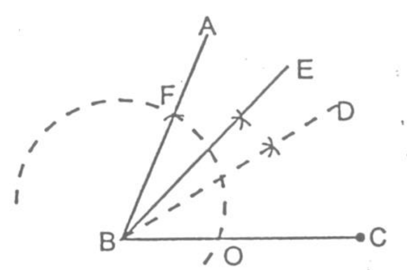
(i) Draw $\angle PBC = 90^0$

(ii) Draw AB which bisects angle PBC,

Then, $\angle ABC = 45^0$

Alternative Method :

STEPS :



(i) Construct $\angle ABC = 60^0$

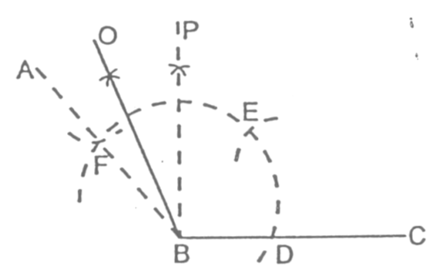
(ii) Draw BD, the bisector of angle ABC.

(iii) Draw BE, the bisector of angle ABD.

Then, $\angle EBC = 45^0$

To Construct an Angle of 105^0 :

STEPS :



(i) Construct $\angle ABC = 120^0$ and $\angle PBC = 90^0$

(ii) Draw BO, the bisector of $\angle ABP$.

Then, $\angle OBC = 105^0$

To Construct an Angle of 150^0.

STEPS :

(i) Draw line segment BC of any suitable length. Produce CB upto any point O.

(ii) With B as centre, draw an arc (with any suitable radius) which buts OC at points D and E.

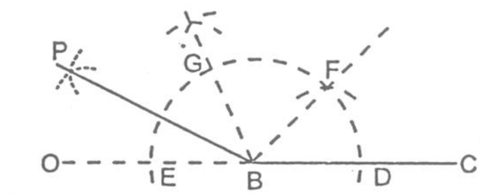
(iii) With D as centre, draw an arc of the same radius, as taken in step 2, which cuts the first arc at point F.

(iv) With F as centre, draw one more arc of the same radius, staken in step 2, which cuts the first arc at point G.

(v) Draw PB, the bisector of angle EBG.

Now $\angle FBD = \angle GBF = \angle EBG = 60^0$

Then, $\angle PBC = 150^0$



To Construct an Angle of 135^0.

STEPS :

(i) Construct $\angle PBC = 150^0$ and $\angle GBC = 120^0$

(ii) Construct BQ, the bisector of angle PBG.

Then, $\angle QBC = 135^0$