CIRCLES

PERPENDICULAR FROM THE CENTRE TO A CHORD

PERPENDUCULAR FROM THE CENTRE TO A CHORD

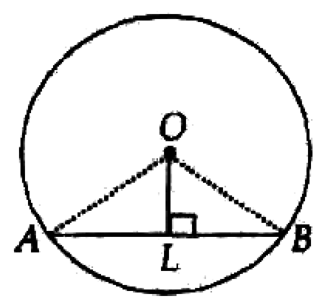
THEOREM 3 :

The perpendicular from the centre of a circle to a chord bisects the chord.

Given : A chord AB of a circle C(O, r) and OL $\perp$ AB.

To Prove: LA = LB

Construction : Join OA and OB.



Proof :

In the right Δs OLA and OLB, we have

OA = OB	[Each equal to r]
OL = OL	[Common]
$\angle$OLA = $\angle$OLB	[Equal to 90º]
$\therefore$ ΔOLA $\cong$ ΔOLB	[By RHS-congruence]
Hence, LA = LB	[C.P.C.T.]

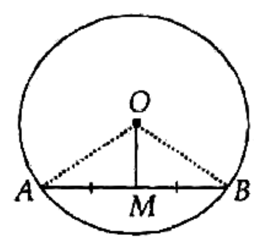
THEOREM 4 :

(Converse of Theorem 3) The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

Given : M is the midpoint of the chord AB of a circle C(O, r).

To Prove : OM $\perp$ AB.

Construction : Join OA and OB.



Proof :

In ΔOMA and ΔOMB, we have

OA = OB	[Each equal to r]
OM = OM	[Common]
MA = MB	[Given]
ΔOMA $\cong$ ΔOMB	[By SSS-congruence].

$\angle OMA = \angle OMB$ ....(1) [By c.p.c.t.]

Now, $\angle OMA + \angle OMB = 180^\circ$ [Linear pair]

$\Rightarrow$ $2\angle OMA = 180^\circ$ [By (1)]

$\Rightarrow$ $\angle OMA = 90^\circ$

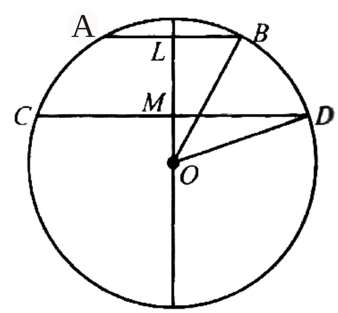
Hence, $OM \perp AB$.

Ex.1 Two chords AB and CD of lengths 6 cm, 12 cm respectively of a circle are parallel. If the $\perp$ distance between AB and CD is 3 cm, find the radius of the circle.

Sol: Here AB = 6 cm $\Rightarrow$ AL = LB = 3 cm

CD = 12 cm $\Rightarrow$ CM = MD = 6 cm

Also, LM = 3 cm. Let OM = x



In right triangle OLB,

$OL^2 + LB^2 = OB^2$ (By pythagoras theorem)

$\Rightarrow$ $(3 + x)^2 + 3^2 = OB^2$ ....(1)

Now in right ΔOMD,

$OM^2 + MD^2 = OD^2$

$\Rightarrow$ $x^2 + 6^2 = OB^2$ ....(2) (since OD = OB = radius)

From (1) & (2),

$(3 + x)^2 + 3^2 = x^2 + 6^2$

$\Rightarrow 9 + 6x = 36 - 9$ or $x = 3$.

From (1), $OB^2 = (3 + 3)^2 + 3^2 = 36 + 9 = 45$

$OB = \sqrt{45} = 3\sqrt{5}$

Hence, radius = $3\sqrt{5}$ cm

Ex.2 In figure, AB $\cong$A Cand O is the centre of the circle. Prove that OA is the perpendicular bisector of BC.

Sol. **Given :** In figure, AB $\cong$AC and O is the centre of the circle.

To Prove : OA is the perpendicular bisector of BC.

Construction : Join OB and OC.

Proof :

$\therefore$ $AB \cong AC$ [Given]

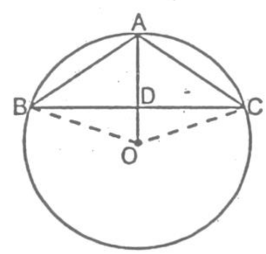
$\therefore$ chord AB = chord AC.

[∵If two arcs of a circle are congruent, then their corresponding chords are equal.]

$\therefore$ $\angle AOB = \angle AOC$....(i)

[∵Equal chords of a circle subtend equal angles at the centre]

In ΔOBC and ΔOCD,



$\angle DOB = \angle DOC$ [From (1)]

$OB = OC$ [Radii of the same circle]

$OD = OD$ [Common]

$\therefore$ $\Delta OBD \cong \Delta OCD$ [By SAS]

$\therefore$ $\angle ODB = \angle ODC$ ....(ii) [By cpctc]

And $BD = CD$ ...(ii) [By cpctc]

But $\angle BDC = 180^0$

$\therefore$ $\angle ODB + \angle ODC = 180^0$

$\Rightarrow$ $\angle ODB + \angle ODB = 180^0$ [From equation (ii)]

$\Rightarrow$ $2\angle ODB = 180^0$

$\Rightarrow$ $\angle ODB = 90^0$

$\therefore$ $\angle ODB = \angle ODC = 90^0$ ....(iv) [From (ii)]

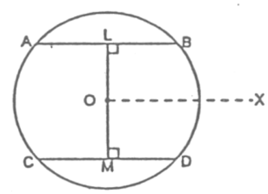
So, by (iii) and (iv), OA is the perpendicular bisector of BC. **Hence Proved.**

Ex.3 Prove that the line joining the mid-points of the two parallel chords of a circle passes through the centre of the circle.

Sol. Let AB and CD be two parallel chords of a circle whose centre is O.

Let l and M be the mid-points of the chords AB and CD respectively. Join PL and OM.

Draw $OX \parallel AB$ or CD.



$\therefore$ L is the mid-point of the chord AB and O is the centre of the circle

$\therefore$ $\angle OLB = 90^0$

[$\because$ The perpendicular drawn from the centre of a circle to chord bisects the chord]

But, $OX \parallel AB$

$\therefore$ $\angle LOX = 90^0$ ......(i)

[$\because$ Sum of the consecutive interior angles on the same side of a transversal is 180^0]

$\therefore$ M is the mid-point of the chord CD and O is the centre of the circle.

$\therefore$ $\angle OMD = 90^0$

[$\because$The perpendicular drawn from the centre of a circle to a chord bisects the chord]

But $OX \parallel CD$ ....(ii)

[$\because$Sum of the consecutive interior angles on the same side of a transversal is 180^0]

$\therefore$ $\angle MOX = 90^0$

From above equations, we get

$\angle LOX + \angle MOX = 90^0 + 90^0 = 180^0$

$\Rightarrow$ $\angle LOM = 180^0$

$\Rightarrow$ LM is a straight line passing through the centre of the circle. **Hence Proved.**

Ex.4 PQ and RS are two parallel chords of a circle whose centre is O and radius is 10 cm. If PQ = 16 cm and RS = 12 cm, find the distance between PQ and RS, if they lie.

(i) on the same side of the centre O.

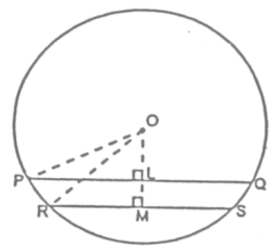
(ii) on opposite sides of the centre O.

Sol. (i) Draw the perpendicular bisectors OL and OM of PQ and RS respectively.

$\therefore$ PQ $\parallel$ RS

$\therefore$ OL and OM are in the same line.

$\Rightarrow$ O, L and M are collinear.



Join OP and OR.

In right triangle OLP,

$OP^2 = OL^2 + PL^2$ [By Pythagoras Theorem]

$\Rightarrow (10)^2 = OL^2 + \left(\frac{1}{2}\times pq\right)^2$

[$\therefore$ The perpendicular drawn from the centre of a circle to a chord bisects the chord]

$\Rightarrow 100 = OL^2 + \left(\frac{1}{2}\times 16\right)^2$

$\Rightarrow 100 = OL^2 + (8)^2$

$\Rightarrow 100 = OL^2 + 64$

$\Rightarrow OL^2 = 100 - 64$

$\Rightarrow OL^2 = 36 = (6)^2$

$\Rightarrow OL = 6$ cm

In right triangle OMR,

$OR^2 = OM^2 + RM^2$ [By Pythagoras Theorem]

$\Rightarrow\ OR^2 = OM^2 + \left(\frac{1}{2}\times RS\right)^2$

[∵The perpendicular drawn from the centre of a circle to a chord bisects the chord]

$\Rightarrow\ (10)^2 = OM^2 + \left(\frac{1}{2}\times 12\right)^2$

$\Rightarrow\ (10)^2 = OM^2 + (6)^2$

$\Rightarrow\ OM^2 = (10)^2 - (6)^2 = (10 - 6)(10 + 6) = (4)(16) = 64 = (8)^2$

$\Rightarrow\ OM = 8$ cm

$\therefore\ LM = OM - OL = 8 - 6 = 2$ cm

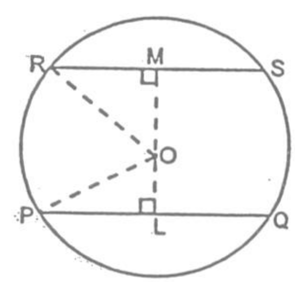
Hence, the distance between PQ and RS, if they lie on he same side of the centre O, is 2 cm.

(ii) Draw the perpendicular bisectors OL and OM of PQ and RS respectively.

$\therefore\ PQ \parallel RS$

$\therefore$ OL and OM are in the same line

$\Rightarrow$ L, O and M are collinear.



Join OP nd OR.

In right triangle OLP,

$OP^2 = OL^2 + PL^2$ [By Pythagoras Theorem]

$\Rightarrow\ OP^2 = OL^2 + \left(\frac{1}{2}\times pQ\right)^2$

[∵The perpendicular drawn from the centre of a circle to a chord bisects the chord]

$\Rightarrow\ (10))^2 = OL^2 + \left(\frac{1}{2}\times 16\right)^2$

$\Rightarrow\ 100 = OL^2 + (8)^2$

$\Rightarrow\ 100 = OL^2 + 64$

$\Rightarrow\ OL^2 = 100 - 64$

$\Rightarrow\ OL^2 = 36 = (6)^2$

$\Rightarrow\ OL = 6$ cm

In right triangle OMR,

$OR^2 = OM^2 + RM^2$ [By Pythagoras Theorem]

$\Rightarrow\ OR^2 = OM^2 + \left(\frac{1}{2}\times 12\right)^2$

[$\because$The perpendicular drawn from the centre of a circle to a chord bisects the chord]

$\Rightarrow\ (10)^2 = OM^2 + \left(\frac{1}{2}\times RS\right)^2$

$\Rightarrow\ (10)^2 = OM^2 + (6)^2$

$\Rightarrow\ OM^2 = (10)^2 - (6)^2 = (10 - 6)(10 + 6) = (4)(16) = 64 = (8)^2$

$\Rightarrow\ OM = 8$ cm

$\therefore\ LM = OL + OM = 6 + 8 = 14$ cm

Hence, the distance between PQ and RS, if they lie on the opposite side of the centre O, is 14 cm.