CIRCLES

EQUAL CHORDS AND THEIR DISTANCES FROM THE CENTRE

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THEOREM 5 :

Equal chords of a circle are equidistant from the centre.

Given : A circle C(O, r) in which chord AB = chord CD, OL ⊥ AB and OM ⊥ CD.

To Prove : OL = OM

Construction : Join OA and OC.

Proof :

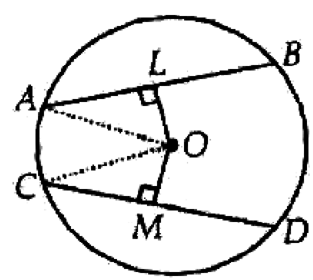
We know that the perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore$ AL = $\frac{1}{2}$ AB and CM = $\frac{1}{2}$ CD.

Since, AB = CD (Given)

$\Rightarrow$ $\frac{1}{2}$ AB = $\frac{1}{2}$ CD

$\Rightarrow$ AL = CM



Now, in ΔOLA and ΔOMC, we have

AL = CM [By (1)]

OA = OC [Each equal to r]

∠OLA = ∠OMC [Each equal to 90º]

∴ΔOLA ≅ ΔOMC [By RHS-congruence]

So, OL = OM [By c.p.c.t.]

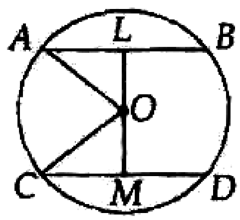
Hence, AB and CD are equidistant from O.

THEOREM 6 :

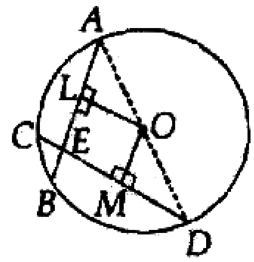
Chords equidistant from the centre of a circle are equal in length. (Converse of Theorem 5)

Given : Circle C(O, r), OL ⊥ AB, OM ⊥ CD, OL = OM

To Prove : AB = CD



Case I (Fig. 1)



Case II (Fig. 2)

Proof :

Case I : AB | | CD

In fig. (1), AB | | CD. Join OA and OC

In ΔOAL and ΔOCM,

OA = OC [Radius]

OL = OM [Given]

$\angle$OLA = $\angle$OMC [Each 90º]

$\Rightarrow$ ΔOAL $\cong$ ΔOCM [By RHS congruency]

$\Rightarrow$ AL = CM [By c.p.c.t.] ....(1)

Since perpendicular from centre bisects the chord

$\Rightarrow$ 2AL = AB ; 2CM = CD ....(2)

From (1) and (2), we get

AB = CD

Case II : AB and CD are intersecting at E.

$\therefore$ Join OA and OD

ΔOAL $\cong$ ΔODM [Same as case I]

$\Rightarrow$ AL = DM [By c.p.c.t.] ....(3)

Since perpendicular from centre bisects the chord

2AL = AB; 2DM = CD ....(4)

From (3) and (4), we get

AB = CD

$\therefore$ From both case I and case II, we get

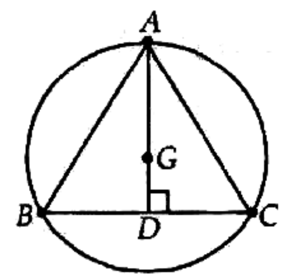
AB = CD

Ex.1 An equilateral triangle of side 9 cm is incribed in a circle. Find the radius of the circle.

Sol: Let ABC be an equilateral triangle of side 9 cm and let AD be one of its medians. Let G be the centoroid of ΔABC. Then, AG : GD = 2 : 1. We know that in a equilateral triangle centroid coincides with the circumcentre. Therefore, G is the centre of the circumcircle with circumradius GA.

Also, G is the centre and GD $\perp$ BC. Therefore, BD = CD = 4.5 cm.

In right triangle ADB, we have



$AB^2 = AD^2 + DB^2$

$\Rightarrow \quad 9^2 = AD^2 + (4.5)^2$

$\Rightarrow \quad AD = \sqrt{81 - \frac{81}{4}} = \frac{9\sqrt{3}}{2}$ cm

$\therefore \quad$ Radius AG $= \frac{2}{3}$ AD $= 3\sqrt{3}$ cm.

Ex.2 AB and CD are equal chords of a circle whose centre is O. When produced, these chords meet at E. Prove that EB = ED.

Sol. **Given :** AB and CD are equal chords of a circle whose centre is O. When produced, these chords meet at E.

To Prove : EB = ED.

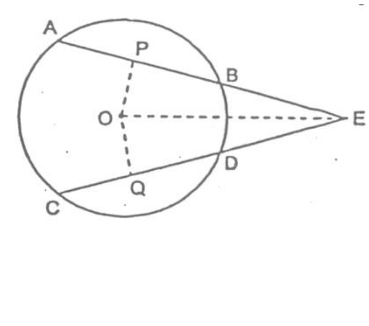
Construction : From O draw OP $\perp$ AB and OQ $\perp$ CD. Join OE.

Proof :

$\therefore$ AB = CD [Given]

$\therefore$ OP = OQ [$\therefore$Equal chords of a circle are equidistant from the centre]

Now in right tingles OPE and OQE,



OE = OE [Common]

Side OP = Side OQ [Proved above]

$\therefore \quad \Delta OPE \cong \Delta OQE$ [By RHS]

$\therefore$ OE = QE [By cpctc]

$\Rightarrow$ PE - $\frac{1}{2}$AB = QE - $\frac{1}{2}$CD [$\therefore$ AB = CD (Given)]

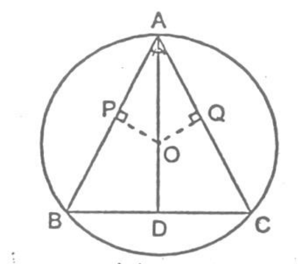
$\Rightarrow$ PE - PB = QE - QD

$\Rightarrow$ EB = ED. **Hence Proved.**

Ex.3 Bisector AD of $\angle BAC$ of ΔABC passed through the centre O of the circumcircle of ΔABC. Prove that AB = AC.

Sol. **Given :** Bisector AD of $\angle BAC$ of ΔABC passed through the centre O of the circumcircle of ΔABC,

To Prove : AB = AC.



Construction : Draw OP $\perp$ AB and OQ $\perp$ AC.

Proof :

In ΔAPO and ΔAQO,

$\angle OPA = \angle OQA$ [Each = 90^0 (by construction)]

$\angle OAP = \angle OAQ$ [Given]

OA = OA [Common]

$\therefore$ $\Delta APO \cong \Delta AQO$ [By ASS cong. prog.]

$\therefore$ OP = OQ [By cpctc]

$\therefore$ AB = AC. [$\because$ Chords equidistant from the centre are equal]

Hence Proved.

Ex.4 AB and CD are the chords of a circle whose centre is O. They intersect each other at P. If PO be the bisector of $\angle APD$, prove that AB = CD.

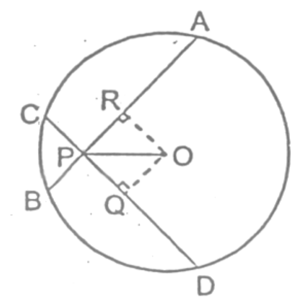
OR

In the given figure, O is the centre of the circle and PO bisect the angle APD. prove that AB = CD.

Sol. **Given :** AB and CD are the chords of a circle whose centre is O. They interest each other at P. PO is the bisector of $\angle APD$.

To Prove : AB = CD.

Construction : Draw OR $\perp$ AB and OQ $\perp$ CD.



Proof :

In ΔOPR and ΔOPQ,

$\angle OPR = \angle OPQ$ [Given]

$OP = OP$ [Common]

And $\angle ORP = \angle OQP$ [Each = 90^0]

$\therefore$ $\Delta ORP \cong \Delta OPQ$ [By AAS]

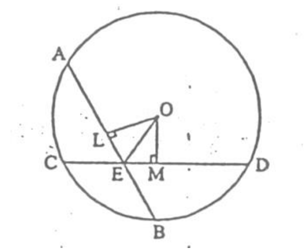
$\therefore$ $OR = OQ$ [By cpctc]

$\therefore$ $AB = CD$

[$\because$Chords of a circle which are equidistant from the centre are equal]

Ex.5 Chords AB and CD of a circle with centre O, intersect at a point E. If OE objects $\angle AED$. Prove that AB = CD.

Sol. In ΔOLE and ΔOME



$\angle OLE = \angle OME$ [90^0 each]

$\angle LEO = \angle MEO$ [Given]

And $OE = OE$ [Common]

$\therefore \Delta OLE \cong \Delta OME$ [By AAS Criteria]

$\Rightarrow OL = OM$ [By cpctc]

This chords AB and CD are equidistant from centre. But we know that only equal chords are equidistant from centre.

$\Rightarrow AB = DC$