CIRCLES

CYCLIC QUADRILATERAL

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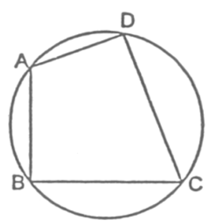
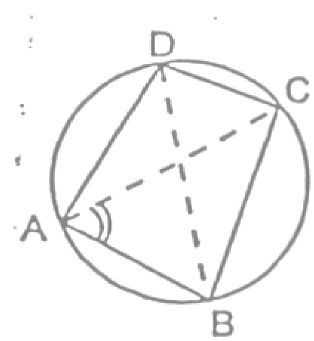
A quadrilateral ABCD is called cyclic if all the four vertices of it lie on a circle.



Theorem-7 :

The sum of either pair of opposite angles of a cyclic quadrilateral is 180^0

Given : A cyclic quadrilateral ABCD.



To Prove : $\angle A + \angle C = \angle B + \angle D = 180^0$

Construction : Join AC and BD.

Proof :

$\angle ACB = \angle ADB$ [Angles of same segment]

And $\angle BAC = \angle BDC$ [Angles of same segment]

$\therefore \angle ACB + \angle BAC = \angle ADB + \angle BDC = \angle ADC.$

Adding $\angle ABC$ to both sides, we get

$\angle ACB + \angle BAC + \angle ABC = \angle ADC + \angle ABC.$

The left side being the sum of three angles of ΔABC is equal to 180^0.

$\therefore$ $\angle ADC + \angle ABC = 180^0$

i.e., $\angle D + \angle B = 180^0$

$\therefore$ $\angle A + \angle C = 360^0$-$(\angle B + \angle D) = 180^0$ [$\therefore$ $\angle A + \angle B + \angle C + \angle D = 360^0$]

Hence Proved.

Corollary :

If the sum of a pair of opposite angles of a quadrilateral is 180^0, then quadrilateral is cyclic.

Ex.1 ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70^0$, $\angle BAC$ is 30^0, find $\angle BCD$. Further, if B = BC, find $\angle ECD$.

Sol. $\angle CDB = \angle BAC = 30^0$ ...(i)

[Angles in the same segment of a circle are equal]

$\angle DBC = 70^0$ ....(ii)

In ΔBCD,

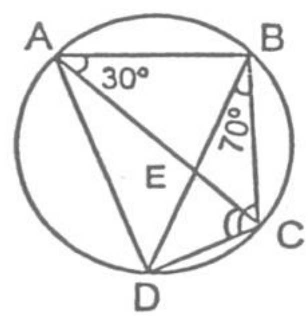
$\angle BCD + \angle DBC + \angle CDB = 180^0$ [Sum of all he angles of a triangle is 180^0]

$\Rightarrow$ $\angle BCD + 70^0 + 30.0 = 180^0$ [Using (i) and (ii)

$\Rightarrow$ $\angle BCD + 100^0 = 180^0$

$\Rightarrow$ $\angle BCD = 180^0 - 100^0$

$\Rightarrow$ $\angle BCD = 80^0$ ...(iii)



In ΔABC,

AB = BC

$\therefore$ $\angle BCA = \angle BAC = 30^0$ ...(iv)

[Angles opposite to equal sides of a triangle are equal]

Now, $\angle BCD = 80^0$ [From (iii)]

$\Rightarrow \angle BCA + \angle ECD = 80^0$

$\Rightarrow 30^0 + \angle ECD = 80^0$

$\Rightarrow \angle ECD = 80^0 - 30^0$

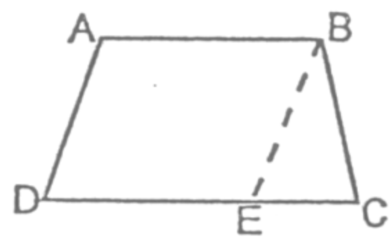
$\Rightarrow \angle ECD = 50^0$

Ex.2 If the nonparallel side of a trapezium are equal, prove that it is cyclic.

Sol. **Given :** ABCD is a trapezium whose two non-parallel sides AB and BC are equal.

To Prove : Trapezium ABCD is a cyclic.

Construction : Draw BE $\parallel$ AD.



Proof :

$\therefore$ AB $\parallel$ DE [Given]

AD $\parallel$ BE [By construction]

$\therefore$ Quadrilateral ABCD is a parallelogram.

$\therefore \angle BAD = \angle BED$ ....(i) [Opp. angles of a $\parallel^{gm}$]

And, AD = BE ....(ii) [Opp. sides of a $\parallel^{gm}$]

But AD = BC ...(iii) [Given]

From (ii) and (iii),

BE = BC

$\therefore \angle BEC = \angle BCE$ ....(iv) [Angles opposite to equal sides]

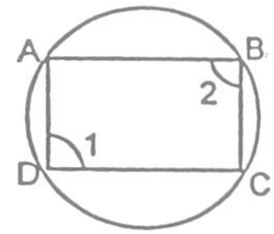
$\angle BEC + \angle BED = 180^0$ [Linear Pair Axiom]

$\Rightarrow \angle BCE + \angle BAD = 180^0$ [From (iv) and (i)]

$\Rightarrow$ Trapezium ABCD is cyclic.

[$\therefore$ If a pair of opposite angles of a quadrilateral 180^0, then the quadrilateral is cyclic] **Hence Proved.**

Ex.3 Prove that a cyclic parallelogram is a rectangle.



Sol. **Given :** ABCD is a cyclic parallelogram.

To Prove : ABCD is a rectangle.

Proof :

$\therefore$ ABCD is a cyclic quadrilateral

$\therefore$ $\angle 1 + \angle 2 = 180^0$ ....(i)

[$\therefore$ Opposite angles of a cyclic quadrilateral are supplementary]

$\therefore$ ABCD is a parallelogram

$\therefore$ $\angle 1 = \angle 2$ ...(ii) [Opp. angles of a $\|$ gm]

From (i) and (ii),

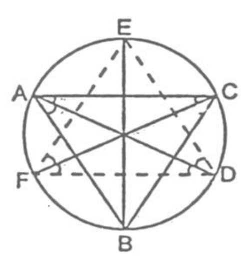
$\angle 1 = \angle 2 = 90^0$

$\therefore$ $\|^{gm}$ ABCD is a rectangle. **Hence Proved.**

Ex.4 Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are

$90^0 - \frac{1}{2}A, 90^0 - \frac{1}{2}B$ and $90^0 - \frac{\angle C}{2}$.

Sol. **Given :** Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively.



To Prove : The angles of the ΔDEF are $90^0 - \frac{\angle A}{2}, 90^0 - \frac{\angle B}{2}$ and $90^0 - \frac{C}{2}$ respectively.

Construction : Join DE, EF and FD.

Proof :

$\angle FDE = \angle FDA + \angle EDA = \angle FCA + \angle EBA$

[$\therefore$ Angles in the same segment are equal]

$= \frac{1}{2}\angle C + \frac{1}{2}\angle B$

$\Rightarrow \angle D = \frac{\angle C + \angle B}{2} = \frac{180^0 - \angle A}{2}$ [$\therefore$ In ΔABC, $\angle A + \angle B + \angle C = 180^0$]

$\Rightarrow \angle D = 90^0 - \frac{\angle A}{2}$

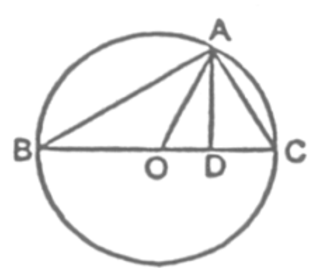
Similarly, we can show that

$\angle E = 90^0 - \frac{\angle B}{2}$

And $\angle F = 90^0 - \frac{\angle C}{2}$. **Hence Proved.**

Ex.5 Find the area of a triangle, the radius of whose circumcircle is 3 cm and the length of the altitude drawn from the opposite vertex to the hypotenuse is 2 cm.

Sol. We know that the hypotenuse of a right angled triangle is the diameter of its circumcircle.



$\therefore$ $BC = 2OB = 2 \times 3 = 6$ cm

Let, $AD \perp BC$

$AD = 2$ cm [Given]

$\therefore$ Area of $\Delta ABC = \frac{1}{2}(BC)(AD) = \frac{1}{2}(6)(2)$

$= 6\ cm^2$. **Ans.**