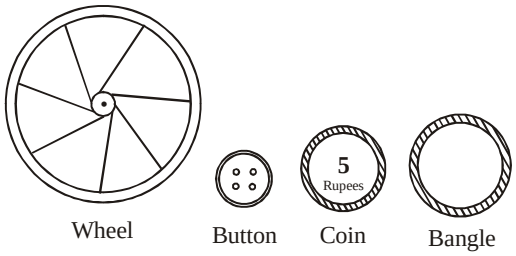
CIRCLES

BASIC TERM RELATED TO CIRCLE

INTRODUCTION

We may have come across many objects in daily life, which are round in shape, such as wheels of a vehicle, bangles, coins of 50 p, Rs.1, Rs.5, buttons of shirts and etc. In a clock you might have observed that the second's hand goes round the dial of the clock rapidly and its tip moves in a round path. This path traced by the tip of the second's hand is called a circle. In this chapter, we will study about circles, other related terms and some properties of a circle.



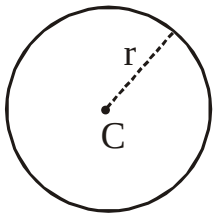
TERMS RELATED TO CIRCLE

Definition of Circle:

The locus of a point which moves in a plane so that its distance from a fixed point in that plane remains constant is called a circle.

In other words, a circle is the set of all those points in a plane each of which is at a constant distance from a fixed point in the plane.

The fixed point is called the centre and the constant distance is called the radius of the circle.

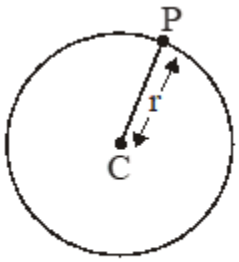


The radius of a circle is always positive. The adjoining figure shows a circle with C as its centre and r as its radius.

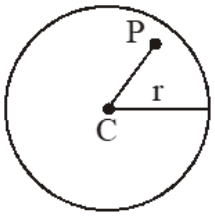
A circle divides the plane on which it lies into three parts. They are

(i) The circle:

A point P lies on the circle if and only if its distance from the centre of the circle is equal to the radius of the circle. See fig (i)



(I)



(II)

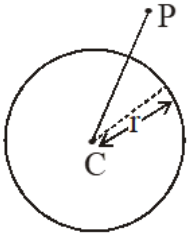
(ii) Interior of a circle:

A point P lies inside a circle if and only if its distance from the centre of the circle is less than the radius of the circle. In the figure(ii), CP < r, therefore, P lies inside the circle.

The set of all points P of the plane such that CP < r form the interior of the circle.

(iii) Exterior of a circle:

A point P lies outside a circle if and only if its distance from the centre of the circle is greater than the radius of the circle.



(III)

In the above figure, CP > r, therefore P lies outside the circle.

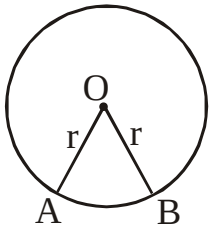
The set of all points P of the plane such that CP > r form the exterior of the circle.

1. Circular Region:

The set of all points P of the plane which either lie on the circle or inside the circle form the circular region.

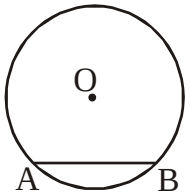
2. Radius:

A line segment joining the centre and a point on the circle is called its radius. The plural of radius is radii. In the given figure, OA and OB are radii of circle C (o, r).



3. Chord:

Line segment joining any two points on a circle is called a chord of the circle. In figure, AB is a chord.

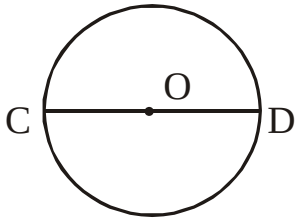


Note : The longer chord is nearer to the centre.

4. Diameter:

A chord which passes through the centre of a circle is called a diameter of the circle. In the figure, CD is a diameter. A diameter divides a circle into two equal parts, each part is called a semicircle.

Length of diameter = 2 × radius.



Note:

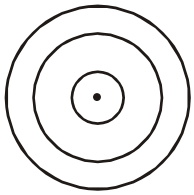
(i) A circle has infinite number of chords.

(ii) The longest chord is the diamter of a circle.

(iii) A circle also has infinite number of diameters.

5. Concentric circles:

Circles having the same centre but with different radii are said to the concentric circles.



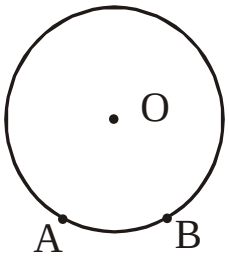
6. Arc of a circle:

The arc of a circle is a portion of cirumference of a circle.

The arc of a circle is denoted by the symbol ''

In the figure, denotes the arc AB of the circle with centre O.

Arc of a circle is divided into following categories:



(i) Circumference:

The whole arc of a circle is called the circumference of the circle. The length of the circumference of a circle is the length of its whole arc.

Circumference of the circle $= 2\pi r$, (r is the radius of circle)

$= \pi \times d$, (d is the diameter of circle)

(ii) Semicircle:

One half of the whole arc of a circle is called a semicircle of the circle. A diameter of a circle divides circle into two equal arcs. Each of these two arcs is called a semicircle.

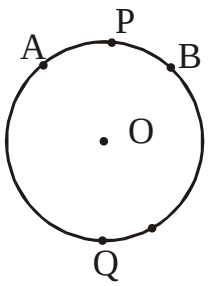
Circumference of semicircle $= \pi r + d = \pi r + 2r$

$= (\pi+2)r$ units.

(iii) Minor arc & major arc:

If the arc is less than a semicircle, then it is called a minor arc.

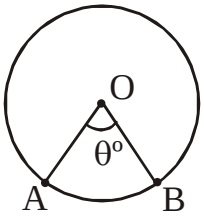
If the arc is greater than a semicircle then it is called a major arc. If we move from A to B in the clockwise direction, it is minor arc AB. But if we move from A to B in the anti clockwise direction, it is major arc AB.



To avoid this confusion we sometimes put a point between the end points of the arc and thus in figure APB is a minor arc and AQB is a major arc.

Note : $\widehat{AB}$ will stand for minor arc AB untill and otherwise stated.

7. Degree measure of an arc:



Let AB be an arc of a circle with centre O.

If $\angle AOB = \theta^\circ$ then degree measure of $\widehat{AB} = \theta^\circ$ or

$m(\widehat{AB}) = \theta^\circ \quad \angle AOB = \theta^\circ$

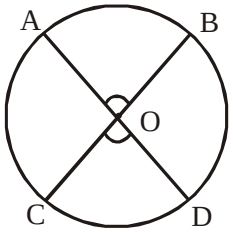
If $m(\widehat{AB}) = \theta^\circ$ then $m(\widehat{BA}) = (360 - \theta)^\circ$

- The degree measure of a circle is 360°.
- An arc whose degree measure is less than 180° is called a **minor arc** of the circle.
- An arc whose degree measure is greater than 180° is called a **major arc** of the circle.
- Relation between the length of an arc and its degree measure–

  Length of an arc $\propto$ degree measure of arc central angle.

  length of arc $= 2\pi r \times \frac{\theta}{360}$

Congruent arcs:



Two arcs $\widehat{AB}$ and $\widehat{CD}$ of a circle are said to be congruent if they have the same degree measures.

$\widehat{AB} = \widehat{CD}$

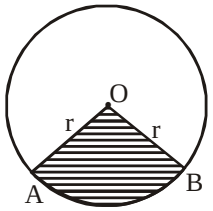
$\Leftrightarrow \quad m(\widehat{AB}) = m(\widehat{CD})$

$\Leftrightarrow \quad \angle AOB = \angle COD$

8. Sector of a circle:

The region enclosed by an arc of a circle and its two bounding radii is called a sector of the circle.

In the fig. OABO is the sector of the circle.



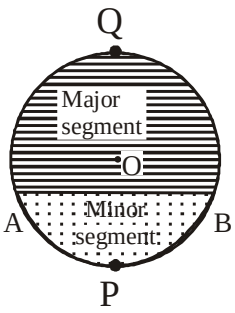
The sector corresponding to the minor arc is called minor sector and the sector corresponding to major arc is called major sector.

Minor sector is shaded.

Area of sector $= \pi r^2 \times \frac{Degree\ measure\ arc}{360°}$

9. Segment of a circle:

A chord of a circle divides a circle into two regions which are called segment of the circle.



The segment containing the minor arc is called the minor segment.

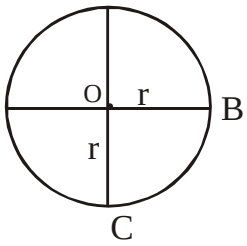
Thus APBA is the minor segment of the circle C (O,r) and the segment containing the major arc is called the major segment. Thus AQBA is the major segment of the circle C (O, r).

Area of segment = Area of sector - $\frac{1}{2} r^2 \sin\theta$

10. Quadrant of a circle:

One fourth of a circle is called a quadrant.

In the fig. OBCO is a quadrant of the circle C (O,r)



11. Congruent circles:

Two circles C (O,r) and C (O,s) are said to be congruent only when r = s,

i.e. Two circles are congruent if and only if they have equal radii.