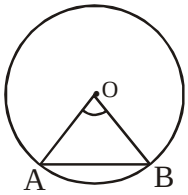
CIRCLES

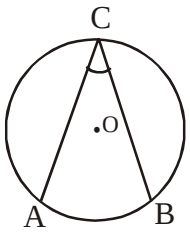
ANGLE SUBTENDED BY A CHORD AT A POINT

Angle subtended by a chord at the centre:

is the angle formed at the centre of the circle when the end points of the chord are joined to the centre. In fig. AB is a chord of the circle. AO and BO are radii of the circle, then ∠AOB is the angle subtended by the chord at the centre of the circle.



Angle subtended by the chord at a point on the circle:



is the angle formed by joining the end points of the chord to a point on the circle.

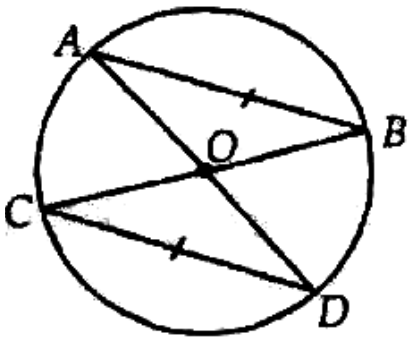
In figure, ∠ACB is the angle subtended by the chord AB at the point C of the circle.

THEOREM 1 :

Equal chords of a circle subtend equal angles at the centre.

Given : A circle C(O, r) in which chord AB = chord CD.

To Prove : ∠AOB = ∠COD



Proof :

In ΔAOB and ΔCOD, we have

OA = OC	[Each equal to r]
OB = OD	[Each equal to r]
AB = CD	[Given]
ΔAOB ≅ ΔCOD	[By SSS-congrunce]
Hence, ∠AOB = ∠COD	[C.P.C.T.]

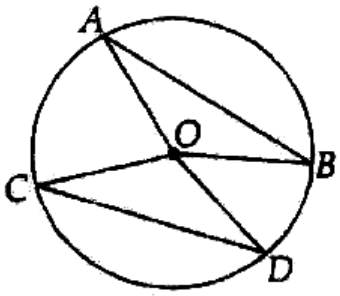
THEOREM 2 :

(Converse of Theorem 1) If the angles subtended by two chords at the centre of a circle are equal then the chords are equal.

Given : A circle C(O, r) in which AB and CD are the chords such that $\angle AOB = \angle COD$.

To prove : $AB = CD$

Proof :



In ΔAOB and ΔCOD, we have

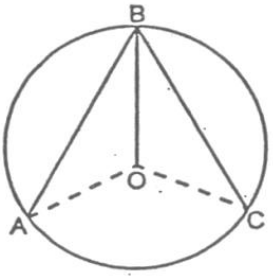
$OA = OC$	[Each equal to r]
$OB = OD$	[Each equal to r]
$\angle AOB = \angle COD$	[Given]
$\Delta AOB \cong \Delta COD$	[By SAS-congruence]
Hence, $AB = CD$	[C.P.C.T.]

Ex.1 In figure, $AB = CB$ and O is the centre of the circle. Prove that BO bisects $\angle ABC$.

Sol. **Given :** In figure, $AB = CB$ and O is the centre of the circle.

To Prove : BO bisects $\angle ABC$.

Construction : Join OA and OC.



Proof :

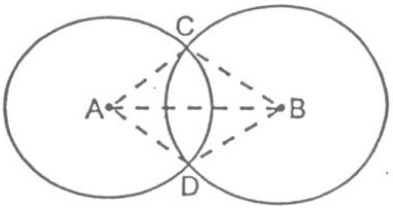
In ΔOAB and ΔOCB,

$OA = OC$	[Radii of the same circle]
$AB = CB$	[Given]
$OB = OB$	[Common]
$\therefore \Delta OAB \cong \Delta OCB$	[By SSS]
$\therefore \angle ABO = \angle CBO$	[By cpctc]
$\Rightarrow$ BO bisects $\angle ABC$.	**Hence Proved.**

Ex.2 Two circles with centres A and B intersect at C and D. Prove that $\angle ACB = \angle ADB$.

Sol. **Given :** Two circles with centres A and B intersect at C and D.

To Prove : $\angle ACB = \angle ADB$.



Construction : Join AC, AD, BC, BD and AB.

Proof :

In ΔACB an ΔADB,

$AC = AD$ [Radii of the same circle]

$BC = BD$ [Radii of the same circle]

$AB = AB$ [Common]

$\therefore \Delta ACB \cong \Delta ADB$ [By SSS]

$\therefore \angle ACB = \angle ADB.$ [By cpctc] **Hence Proved.**