NUMBER SYSTEMS

REPRESENTING IRRATIONAL NUMBER ON THE NUMBER LINE

IRRATIONAL NUMBERS :

A number is an irrational number, if it has a non terminating and non-repeating decimal representations. A number that cannot be put in the form p/q where p, q are integers and q0 is called irrational number.

e.g. $\sqrt{2}, \sqrt{3}, \sqrt{11}$ p etc.

Some Properties of irrational numbers :

(a) The –ve of an irrational number is an irrational number.

(b) The sum of a rational and an irrational number is an irrational number.

(c) The product of a non–zero rational number with an irrational number is always an irrational number.

Ex.1 Insert an irrational number between 2 and 3.

Sol. $\sqrt{2 \times 3} = \sqrt{6}$

Ex.2 Find two irrational number between 2 and 2.5.

Sol. **1st Method :** $\sqrt{2 \times 2.5} = \sqrt{5}$

Since there is no rational number whose square is 5. So $\sqrt{5}$ is irrational..

Also $\sqrt{2 \times \sqrt{5}}$ is a irrational number.

2nd Method :

2.101001000100001.... is between 2 and 5 and it is non-recurring as well as non-terminating.

Also, 2.201001000100001......... and so on.

Ex.3 Find two irrational number between $\sqrt{2}$ and $\sqrt{3}$.

Sol. **1st Method :** $\sqrt{\sqrt{2}\times\sqrt{3}}=\sqrt{\sqrt{6}}=\sqrt[4]{6}$

Irrational number between $\sqrt{2}$ and $\sqrt[4]{6}$

$\sqrt{\sqrt{2}\times\sqrt[4]{6}}=\sqrt[4]{2}\times\sqrt[8]{6}$

2nd Method :

As $\sqrt{2}$ = 1.414213562 ...... and $\sqrt{3}$ = 1.732050808......

As , $\sqrt{3}>\sqrt{2}$ and $\sqrt{2}$ has 4 in the 1st place of decimal while $\sqrt{3}$ has 7 is the 1st place of decimal.

1.501001000100001......., 1.601001000100001...... etc. are in between $\sqrt{2}$ and $\sqrt{3}$

Representing the Square Root of a Positive Number on the Number Line

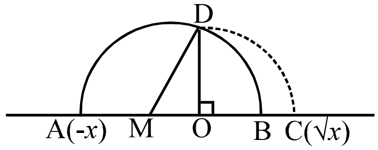
Let x be a positive real number. We will now locate $\sqrt{x}$ on the number line.

Step. 1 : Mark –x on the number line. Let this point be represented by A. Mark 1 unit on the number line. Let this be represented by B.

Step. 2 : Locate the midpoint M of AB.

Step. 3 : With M as the centre and MA or MB as radius draw a semicircle. Since diameter AB = (x + 1) units, MA = MB = ½ (x + 1) units.

Step. 4 : Draw OD perpendicular to AB meeting the semicircle in D. Join MD. Note the DMO is a right triangle with MD = ½ (x+1) units and MO = [½ (x + 1) – 1) units. = ½ (x – 1) units.



Step 5 : Using the Pythagorean theorem, we obtain :

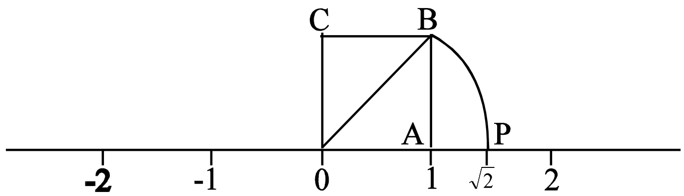
$OD^2 = MD^2 - MO^2 = \frac{1}{4}(x+1)^2 - \frac{1}{4}(x-1)^2 = \frac{1}{4}(4x) = x = OD = \sqrt{x}$

With O as the centre and OD as the radius, draw an arc to meet the number line at C. The point C represents $\sqrt{x}$.

Ex.4 : Locate $\sqrt{2}$ on the number line.

Sol. **Step 1 :** Draw the number line with O representing the number 0 and A representing the number 1.

Step 2 : Construct a square OABC with each side equal to 1 unit.



By the Pythagorean theorem :

$OB^2 = OA^2 + AB^2 = 1^2 + 1^2 = 1 + 1 = 2$

$OB = \sqrt{2}$

Step 3 : With O as centre and OB as radius, draw an arc to meet the number line at point P.

Since $OP = OB = \sqrt{2}$, the point P represents $\sqrt{2}$ on the number line.

Remark : In the same way, we can locate $\sqrt{n}$ for any positive integer n, after $\sqrt{n-1}$ has been located.

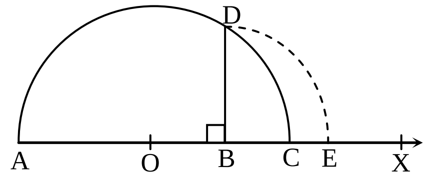
Existence of $\sqrt{n}$ for a positive real number :

The value of $\sqrt{4.3}$ geometrically : -

Draw a line segment AB = 4.3 units and extend it to C such that BC = 1 unit.

Find the midpoint O of AC.

With O as centre and OA a radius, draw a semicircle.



Now, draw $BD \perp AC$, intersecting the semicircle at D. Then, $BD = \sqrt{4.3}$ units.

With B as centre and BD as radius, draw an arc, meeting AC produced at E.

Then, BE = BD = $\sqrt{4.3}$ units