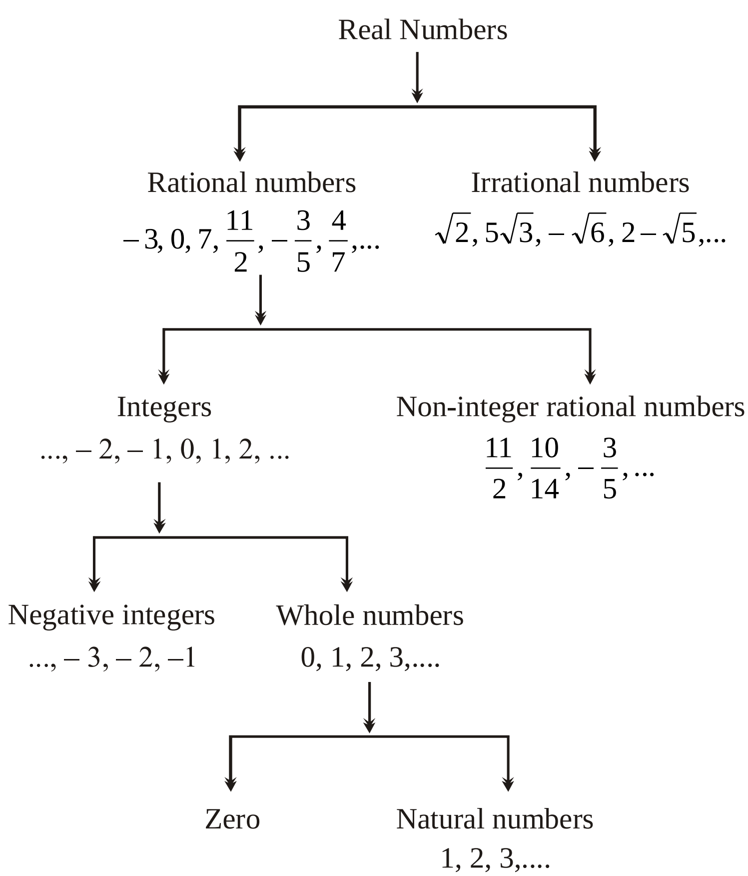
NUMBER SYSTEMS

INTRODUCTION OF NUMBER SYSTEM

INTRODUCTION:

In earlier classes, we have learnt about natural numbers, whole numbers and integers. As we have learnt that the counting numbers 1,2,3, ... etc. are called natural numbers and all the natural numbers together with zero are called whole numbers. In this chapter, we shall introduce the system of rational numbers and we shall also extend out study on real numbers, their decimal representation, representation on the number line and operations on real numbers.

NUMBER SYSTEM:



In Hindu Arabic system we use ten symbols 0,1,2,3,4,5,6,7,8,9 called digits to represent any number. A group of figures, denoting a number is called numeral.

Types of Numbers:

(1) Natural Numbers :

Counting numbers are called natural numbers.

N = {1,2,3,4,........} is a set of all natural numbers.

(2) Whole Numbers :

All counting numbers together with zero form a set of all whole numbers.

W = {0,1,2,3,4 ....} is a set of all whole numbers.

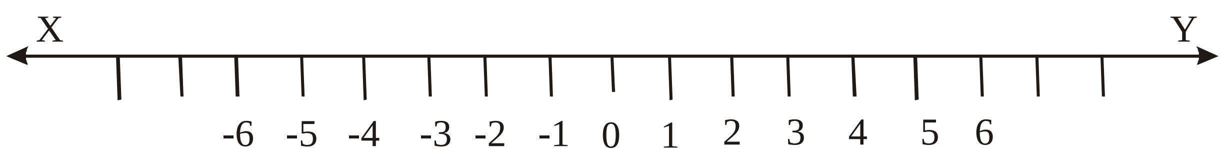
(3) Integers :

All natural numbers, 0 and negative of natural numbers form integers.

I = { .... -4, -3, -2, -1, 0 , 1, 2, 3 ....}

All integers can be represented on the number line.

Number line :



Positive Integers :

On the right hand side of 0, the points at distances of 1 unit, 2 units, 3 units etc. from 0 denote respectively the integers 1,2,3 etc.

Negative Integers :

On the left side of 0, the points at distances of 1 units, 2 units, 3 units etc. from 0 denote respectively the integers –1, – 2, – 3 ... etc.

NOTE :

(i) “0” is neither positive, nor negative.

(ii) Non-negative integers: 0, 1, 2, ...

(iii) Non-positive integers: .... – 3, – 2, – 1, 0,

(iv) Positive integers: 1, 2, 3, ...

(v) Negative integers: .... – 3, – 2, – 1.

RATIONAL NUMBERS:

A number which can be expressed in the form p/q where p,q are integers, and q 0 is called a rational number.

Each integer is a rational number, An integer m can be written as m/1 to put in the form p/q where p,q are integers and q 0 .

Equivalent Rational Numbers

Rational numbers do not have a unique representation. For instance, 2/3 can be : represented by any of the following :

$$\frac{4}{6}, \frac{6}{9}, \frac{10}{15}, \frac{-44}{-66}, \ldots\ldots$$

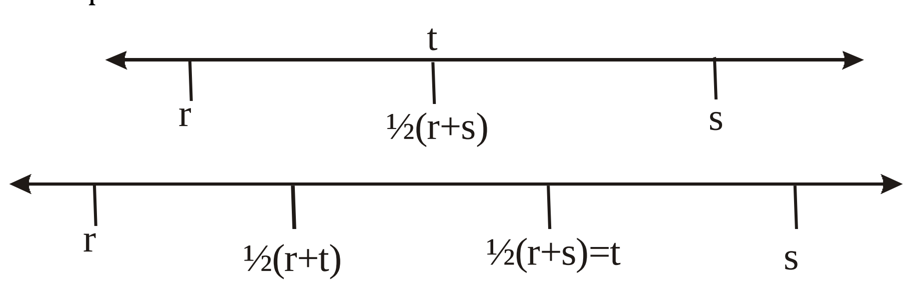
All such numbers are called equivalent rational numbers.

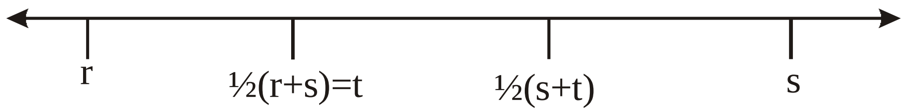
Inserting Rational Number between Two Given Rational Numbers:

Between any two distinct rational numbers x and y, there exists infinitely many rational numbers.

(i) First Method:

To insert two rational numbers between r and s, we first insert the number (r+s) = t(say) and then repeat the procedure with r and t or with t and s.





We may repeat the process any number of times.

(ii) Second Method:

If we wish to write n rational numbers between r and s. We write r and s as fractions, whose denominators are one more than n, the number of rational numbers to be inserted. That is, we write :

$$r = \frac{r'}{n+1} \text{ and } s = \frac{s'}{n+1}$$

Then the desired n rational numbers are : $\frac{r'+1}{n+1}, \frac{r'+2}{n+1}, \frac{r'+n}{n+1}, \ldots\ldots\ldots \frac{r'+n}{n+1}$

Complex numbers:

Complex numbers are imaginary numbers of the form a + ib, where a and b are real numbers and i = $\sqrt{-1}$, which is an imaginary number.

Factors:

A number is a factor of another, if the former exactly divides the latter without leaving a remainder (remainder is zero) 3 and 5 are factors of 12 and 25 respectively.

Multiples:

A multiple is a number which is exactly divisible by another, 36 is a multiple of 2, 3, 4, 9 and 12.

Even Numbers:

All integers which are multiples of 2 are even number (i.e.) 2,4, 6, 8............... are even numbers.

Odd numbers :

All integers which are not multiples of 2 are odd numbers.

Prime and composite Numbers:

All natural numbers which cannot be divided by any number other than 1 and itself is called a prime number. By convention, 1 is not a prime number.

2, 3, 5, 7, 11, 13, 17 .... are prime numbers. Numbers which are not prime are called composite numbers.

Identification Prime Number:

Step 1 : Find approximate square root of given number.

Step 2 : Divide the given number by prime numbers less than approximate square root of number. If given number is not divisible by any of this prime number then the number is prime otherwise not.

Ex.1 571, is it a prime ?

Sol. Approximate square root of 571 = 24.

Prime number < 24 are 2, 3, 5, 7, 11, 13, 17, 19, & 23. But 571 is not divisible by any of these prime numbers so 571 is a prime number.

Ex.2 Is 1 prime or composite number ?

Sol. 1 is neither prime nor composite number

Twin Prime :

The term twin primes is used for a pair of odd prime numbers that differ by two. e.g. 3 and 5 are twin primes.

Co-prime numbers :

If the H.C.F. of the given numbers (not necessarily prime) is 1 then they are known as co-prime numbers. e.g. 5, 6, are co-prime as H.C.F. of (5, 6) = 1.

NOTE:

(i) 1 is neither prime nor composite number.

(ii) 2 is the only prime number which is even.

(iii) Any two consecutive numbers will always be co-prime.

The Absolute Value (or modulus) of a real Number:

If a is a real number, modulus a is written as $|a|$; $|a|$ is always positive or zero.It means positive value of 'a' whether a is positive or negative

$|3| = 3$ and $|0| = 0$, Hence $|a| = a$; if $a = 0$ or $a > 0$ (i.e.) $a \geq 0$

$|-3| = 3 = -(-3)$. Hence $|a| = -a$ when $a < 0$

Hence, $|a| = a$, if $a > 0$; $|a| = -a$, if $a < 0$

Irrational number :

(i) All real numbers are irrational if and only if there decimal representation is non-terminating and non-repeating. e.g. $\sqrt{2}$, $\sqrt{3}$, π............ etc.

(ii) Rational number and irrational number taken together form the set of real numbers.

(iii) If a and b are two real numbers, then either
(i) $a > b$ or (ii) $a = b$ or (iii) $a < b$

(iv) Negative of an irrational number is an irrational number.

(v) The sum of a rational number with an irrational number is always irrational.

(vi) The product of a non-zero rational number with an irrational number is always an irrational number.

(vii) The sum of two irrational numbers is not always an irrational number.

(viii) The product of two irrational numbers is not always an irrational number.

Rational Numbers: $3, 4, \frac{7}{3}, \frac{5}{2}, -\frac{3}{7}$, 2.7, 3.923, 1.427, 1.2343434, etc.

Irrational Numbers: $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \pi$, 1.327185............

Imaginary Numbers : $\sqrt{-2}, \sqrt{-49}, 3i, \left(\frac{5}{7}+\frac{\sqrt{-3}}{8}\right),$ ....

Note: $\pi = 3.14159265358979$............ while $\frac{22}{7} = 3.1428571428$............

$\pi \neq \frac{22}{7}$ but for calculation we can take $\pi \approx \frac{22}{7}$.

Ex.3: Find three rational numbers between – 2 and 5.

Sol. A rational number lying between – 2 and 5 is

$(-2+5) , 2 = 3 , 2 = \frac{3}{2}$ i.e., $-2 < \frac{3}{2} < 5$

Now, a rational number lying between – 2 and $\frac{3}{2}$ is

$$\left(-2+\frac{3}{2}\right),2=\left(\frac{-2\times2+3}{2}\right),2=\left(\frac{-4+3}{2}\right),2=\left(\frac{-1}{2}\right),2=\frac{-1}{2}\times\frac{1}{2}=\frac{-1}{4}$$

A rational number lying between $\frac{3}{2}$ and 5 is

$$\left(\frac{3}{2}+5\right),2=\left(\frac{3+5\times2}{2}\right),2=\frac{13}{2}\times\frac{1}{2}=\frac{13}{4}$$

$$-2<\frac{-1}{4}<\frac{3}{2}<\frac{13}{4}<5$$

Hence, three rational numbers between – 2 and 5 are : $\frac{-1}{4}, \frac{3}{2}, \frac{13}{4}$

Ex.4: Write 2 equivalent rational numbers of $\frac{2}{7}$.

Sol. $\frac{2}{7}=\frac{2\times2}{7\times2}=\frac{4}{14}$

$\frac{2}{7}=\frac{2\times4}{7\times4}=\frac{8}{28}$

Ex.5 Find 4 rational numbers between 2 and 3.

Sol. (i) Write 2 and 3 multipling in N^r and D^r with (4+1).

(ii) i.e. 2 $\frac{2\times(4+1)}{(4+1)}=\frac{1}{5}$ & $3=\frac{3\times(4+1)}{(4+1)}=\frac{15}{5}$

(iii) So, the four required numbers are $\frac{11}{5}, \frac{12}{5}, \frac{13}{5}, \frac{14}{5}$.

Ex.6 Find three rational no's between a and b (a < b).

Sol. $a < b$

$\Rightarrow a + a < b + a$

$\Rightarrow 2a < a + b$

$\Rightarrow a < \frac{a+b}{2}$

Again, $a < b$

$\Rightarrow a + b < b + b.$

$\Rightarrow a + b < 2b$

$\Rightarrow \frac{a+b}{2} < b.$

$\therefore a < \frac{a+b}{2} < b.$

i.e. $\frac{a+b}{2}$ lies between a and b.

Hence 1st rational number between a and b is $\frac{a+b}{2}$.

For next rational number

$$\frac{a+\frac{a+b}{2}}{2} = \frac{\frac{2a+a+b}{2}}{2} = \frac{3a+b}{4} \quad \therefore \quad a < \frac{3a+b}{4} < \frac{a+b}{2} < b.$$

Next, $\frac{\frac{a+b}{2}+b}{2} = \frac{a+b+2b}{2\times 2} = \frac{a+3b}{4}$

$\therefore \quad a < \frac{3a+b}{4} < \frac{a+b}{2} < \frac{a+3b}{4} < b$, and continues like this.

Ex.7 Find 3 rational numbers between $\frac{1}{3}$ & $\frac{1}{2}$.

Sol. **1st Method** $\dfrac{\frac{1}{3}+\frac{1}{2}}{2}=\dfrac{\frac{2+3}{6}}{2}=\dfrac{5}{12}$

$\therefore \frac{1}{3}, \frac{5}{12}, \frac{1}{2}$

$= \dfrac{\frac{1}{3}+\frac{5}{12}}{2}=\dfrac{\frac{4+5}{12}}{2}=\dfrac{9}{24}$

$\therefore \frac{1}{3}, \frac{9}{24}, \frac{5}{12}, \frac{1}{2}$

$= \dfrac{\frac{5}{12}+\frac{1}{2}}{2}=\dfrac{\frac{5}{12}+\frac{6}{12}}{2}=\dfrac{11}{24}$

$\therefore \frac{1}{3}, \frac{9}{24}, \frac{5}{12}, \frac{11}{24}, \frac{1}{2}.$

Verify : $\frac{8}{24}<\frac{9}{24}<\frac{10}{24}<\frac{11}{24}<\frac{12}{24}\left(\text{as}\frac{8}{24}=\frac{1}{3}\&\frac{1}{2}\right)$

2nd Method : Find n rational numbers between a and b ($a < b$).

(i) Find $d = \dfrac{b-a}{n+1}$.

(ii) 1st rational number will be $a + d$.

2nd rational number will be $a + 2d$.

3rd rational number will be $a + 3d$ and so on....

nth rational number is $a + nd$.

Ex.8 Find 5 rational number between $\frac{3}{5}$ and $\frac{4}{5}$

Here, $a=\frac{3}{5}, b=\frac{4}{5} d=\frac{b-a}{n+1}=\frac{\frac{4}{5}-\frac{3}{5}}{5+1}=\frac{1}{5}\times\frac{1}{6}=\frac{1}{30}$.

$1^{st}=a+b=\frac{3}{5}+\frac{1}{30}=\frac{19}{20},$ $2^{nd}=a+2d=\frac{3}{5}+\frac{2}{30},$

$3^{rd}=a+3d=\frac{3}{5}+\frac{3}{30}=\frac{21}{30},$ $4^{th}=a+4d=\frac{3}{5}+\frac{4}{30}=\frac{22}{30},$

$5^{th}=a+5d=\frac{3}{5}+\frac{5}{30}=\frac{23}{30}.$

Perfect Numbers :

If the sum of all factors of a number is twice the number then this number is called perfect number.

If $2^k - 1$ = Prime number, then $(2^k - 1)(2^{k-1})$ is a perfect number.

e.g., 6, 28, etc.

Fractions:

(a) Common fraction: Fractions whose denominator is not 10.

(b) Decimal fraction : Fractions whose denominator is 10 or any power of 10.

(c) Proper fraction: Numerator $<$ Denominator i.e. $\frac{3}{5}$.

(d) Improper fraction: Numerator $>$ Denominator i.e. $\frac{5}{3}$.

(e) Mixed fraction: Consists of integral as well as fractional part i.e. $3\frac{2}{7}$.

(f) Compound fraction: Fraction whose numerator and denominator themselves are fractions. i.e. $\frac{2/3}{5/7}$.

Note: Improper fraction can be written in the form of mixed fractions.