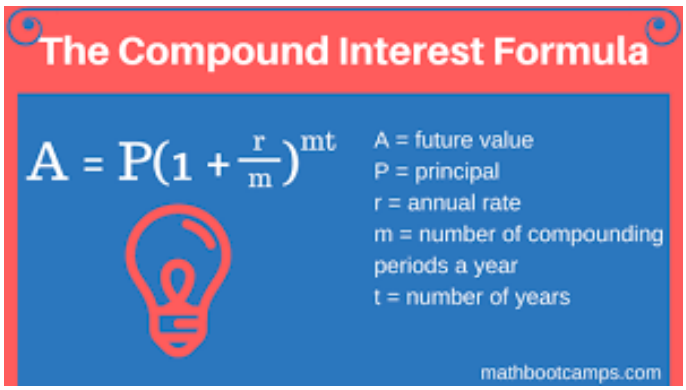
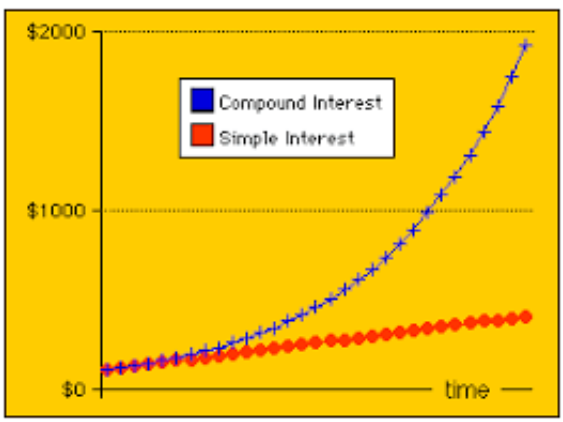
COMPOUND INTEREST

Compound Interest-

When the borrower X and the lender Y agrees to fix up a certain time for example yearly, half yearly or quarterly to settle the previous money, then the difference between the amount and the money borrowed is said to be the Compound Interest and it denoted by C.I. In these calculations, principal for the second unit of time is the amount of first unit of time and so on.



In compound interest, the interest for each period is added to the principle before interest is calculated for the next period. With this method the principle grows as the interest is added to it. This method is mostly used in investments such as savings account and bonds.



Basic Formulas of Compound Interest

If A = Amount
P = Principle
C.I. = Compound Interest
T = Time in years
R = Interest Rate Per Year

$$C.I. = P\left[\left(1 + \frac{R}{100}\right)^{T} - 1\right]$$

$$A = P\left(1 + \frac{R}{100}\right)^{T}$$

Amount Due at the end of the time period is given byf

$$A = p(1 + \frac{r}{100})^{t}$$

Where:
P: Principal (original amount)
R: Rate of Interest (in %)
T: Time period (yearly, half-yearly etc.)

The Compound Interest over the time period T is given by by the formula:

$$A - P = P\left(1 + \frac{r}{100}\right)^{t} - P$$

This can be written as:
Compound Interest= P $\{(1+r/100)^{t} -1\}$

To understand compound interest clearly, let's take an example.

1000 is borrowed for three years at 10% compound interest. What is the total amount after three years?

Year	Principle	Interest (10%)	Amount
1st	1000	100	1100
2nd	1100	110	1210
3rd	1210	121	1331

After three years,

In simple interest, the total amount would be 1300

And in compound interest, the total amount would be 1331.

Shortcut formulas for compound interest-

Rule 1: If rate of interest is R1% for first year, R2% for second year and R3% for third year, then

$$A = P\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1 + \frac{R_3}{100}\right)$$

Rule 2:

If principle = P, Rate = R% and Time = T years then

1. **If the interest is compounded annually:**

$$A = P\left(1 + \frac{R}{100}\right)^T$$

2. **If the interest is compounded half yearly (two times in year):**

$$A = P\left(1 + \frac{R/2}{100}\right)^{2T}$$

3. **If the interest is compounded quarterly (four times in year):**

$$A = P\left(1 + \frac{R/4}{100}\right)^{4T}$$

Rule 3:

If difference between Simple Interest and Compound Interest is given.

- If the difference between Simple Interest and Compound Interest on a certain sum of money for 2 years at R% rate is given then

$$\text{Sum} = \text{Difference}\left(\frac{100}{R}\right)^2$$

- If the difference between Simple Interest and Compound Interest on a certain sum of money for 3 years at R% is given then

$$\text{Sum} = \frac{\text{Difference} \times 100^3}{R^2(300 + R)}$$

Rule 4:

If sum A becomes B in T_1 years at compound interest, then after T_2 years

$$\text{Sum} = \frac{B^{\frac{T_2}{T_1}}}{A^{\frac{T_2}{T_1}} - 1}$$

Look up Table

	4%	5%	10%	20%
1 year $\left(1+\frac{r}{100}\right)$	26/25	21/20	11/10	6/5
2 year $\left(1+\frac{r}{100}\right)^2$	676/625	441/400	121/100	36/25
3 year $\left(1+\frac{r}{100}\right)^3$	17576/15625	9261/8000	1331/1000	216/125

Installment paid with compound interest-

To calculate the installments paid with compound interest, we use the following formula

$$P\left(1+\frac{R}{100}\right)^n = x\left(1+\frac{R}{100}\right)^{n-1} + x\left(1+\frac{R}{100}\right)^{n-2} + x\left(1+\frac{R}{100}\right)^{n-3} + x\left(1+\frac{R}{100}\right)^{n-4} + \cdots$$

Where,

P= Principal

R=rate

n= number of installments

×= Amount of installment

Example:

A sum of Rs. 1275 is borrowed at 4% pa compound interest and paid back in 2 equal annual installments. What is the amount of each installment?

Solution:

Let the value of installment be ×

Equating the amounts

1275×(1.04)2=× +1.04×

×= Rs. 676

Example:

A sum of Rs. 550 is to be repaid in 2 equal annual installments. If rate =20% compounded annually, then the value of each installment will be?

Solution:

Let the value of installment be ×

Equating the amounts

$550*(1.2)^2 = \times + 1.20\times$

$\times$= Rs. 360

Example

A Sum of Rs. 2600, is lent out in two parts S.I. at 10% for 5 yr is equal to S.I. on 2nd part at 9% rate for 6 yr. find the ratio of parts.

Solution :

Given SI1 = SI 2

P1 : P2 = 1/R1T1 : 1/R2T2

= 1/(10*5) : 1/(9:6)

27:25

Rate⇒ Time⇓	5%			10%			15%			20%		
	SI	CI	Diff	SI	CI	Diff	SI	CI	Diff	SI	CI	Diff
1 year	1.05	1.05	0	1.1	1.1	0	1.2	1.2	0	1.2	1.2	0
2 years	1.10	1.1025	0.0025	1.2	1.21	0.01	1.4	1.44	0.04	1.6	1.69	0.09
3 years	1.15	1.1575	0.0075	1.3	1.331	0.031	1.6	1.728	0.128	1.9	2.197	0.297

Example 1:

Maninder took a loan of Rs. 10000 from Prashant. If the rate of interest is 5% per annum compounded annually, find the amount received by Prashant by the end of three years

Solution:

The following is the data given:

Principal, P= 10000

Rate = 5%

Time =3 years

Using the formula for Compound Interest:

$A = P(1+R/100)^t$

So $A= 10000(1+5/100)^3$

$A = 10000(1+1/20)^3$

A = 10000 x 21/20 x 21/20 x21/20 =11576.25

So, the total amount paid by Maninder at the end of third year is Rs.11576.25

Example 2:

Richa gave Rs. 8100 to Bharat at a rate of 9% for 2 years compounded annually. Find the amount of money which she gained as a compound interest from Bharat at the end of second year.

Solution:

Principal value = 8100

Rate = 9%

Time = 2 years

So the total amount paid by Bharat

$= 8100(1+9/100)^2$

=Rs. 9623.61

The question does not probe the amount, rather, it wants to know the CI paid, that the difference between the total amount and original principal.

The Compound Interest = 9623.61 – 8100 = 1523.61

Example 3:

The difference between compound interest and simple interest is 2500 for two years at 2% rate, then find the original sum.

Solution:

Given difference is = 2500

So, Simple Interest = (P X R X T)/100

Compound Interest = $P [(1+R/100)^t – 1]$

So the difference between both of them is

$= P[(1+R/100)^T -1] – PRT/100$

$= P [(1+R/100)^T -1-RT/100]$

$2500 = P [\{(1+2/100)^2-1\}-4/100]$

On simplification this equation, the sum will be = Rs. 6250000

Let's use the shortcut method:

When time given is 2 years, Difference = $P(R/100)^2$

Since the difference given is Rs 2500, we have

$2500 = P (2/100)^2$

$\Rightarrow 2500 = P (1/50)^2$

$\Rightarrow 2500= P (1/2500)$

$\Rightarrow 6250000=P$

So, the sum is Rs.6250000.

Question 1:

An amount of Rs 1000 is borrowed at CI at the rate of 2% per annum. What will be the amount to be paid after 3 years if the interest is compounded annually?

A. 926.24

B. 1248.34

C. 1061.28

D. 1678.34

Answer and Explanation

Option C.

$$A = p(1+\frac{r}{100})^t$$

$$A=1000(1+\frac{2}{100})^3$$

A=1061.208

Question 3:

The difference between the compound interest and the simple interest on a certain sum at 4% rate for 2 years is Rs 100. What will be the amount invested?

A. 45500

B. 52500

C. 62000

D. 62500

Option D

The difference between compound interest and simple interest for two years is given by

$$p\left(\frac{r}{100}\right)^2$$

Hence

$$100 = p\left(\frac{4\times4}{100\times100}\right)$$

$$\Rightarrow 100 = p\left(\frac{1}{625}\right)$$

$$\Rightarrow p = Rs\,62500$$

Question 4:

The difference between the compound interest and the simple interest on a certain sum at 4% rate for 3 years is Rs 100. What will be the amount invested?

A. 20559

B. 25559

C. 16559

D. 28559

Option A

The difference between CI and SI for three years is given by

$$\frac{PR^2(300+R)}{(100)^3}$$

Hence, we have

$$100 = \frac{P\times4^2(300+4)}{100^3}$$

$$\Rightarrow P = \frac{100\times100\times100\times100}{16\times304} = Rs\ 20559.21$$

Question 5:

A sum of money invested at compound interest amounts to Rs. 650 at the end of first year and Rs. 676 at the end of second year. The sum of money is:

A. Rs. 600

B. Rs. 540

C. Rs. 625

D. Rs. 560

Option C

Simple Interest for one year = Compound Interest for one year

Interest on Rs. 650 for 1 year = 676 –650 = Rs. 26

$$SI = \frac{p\times r\times t}{100}$$

$$r = \frac{26}{650}\times100$$

r = 4% per annum

$$A = p(1+\frac{r}{100})^t$$

$$P = \frac{A}{\left[1+\frac{r}{100}\right]^t} = \frac{650}{\left[1+\frac{4}{100}\right]^t}$$

$$\frac{650}{\frac{26}{25}} = 650\times\frac{25}{26} = Rs.625.$$

A sum at a rate of interest compounded yearly becomes Rs. A_1 in n years and Rs. A_2 in (n + 1) years, then

$$P = A_1\left(\frac{A_1}{A_2}\right)^n$$

Example-3:

A sum of money invested at compound interest amounts to Rs. 100 at the end of first year and Rs. 120 at the end of second year. The sum of money is :

Solution:

Simple Interest for one year = compound interest for one year

Interest on Rs. 100 for 1 year = 120-100= Rs. 20

$$SI = \frac{p \times r \times t}{100}$$

$$r = \frac{20}{100} \times 100 = 20\%$$

$$A = p(1+\frac{r}{100})^t$$

$$P = \frac{A}{\left[1+\frac{r}{100}\right]^t} = \frac{100}{\left[1+\frac{20}{100}\right]^1} = \frac{100}{\frac{6}{5}} = Rs\,\frac{500}{6}$$

Alternative Method:

We can use the formula: $P = A_1\left(\frac{A_1}{A_2}\right)^n$.

$$\Rightarrow P = 100 \times \left(\frac{100}{120}\right)^1 = Rs\,\frac{500}{6}$$

If an amount of money grows up to Rs x in t years and up to Rs y in (t+1) years on compound interest, then

$$R\% = \frac{y - x}{x} \times 100$$

Derivation for this result:

Principal + CI for t years = x …… (1)

Principal + CI for (t+1) years= y ……. (2)

(2) – (1) =>CI for last year = y-x

Which is basically the simple interest upon x

$$SI = \frac{p \times r \times t}{100}$$

$$\Rightarrow y - x = \frac{x \times r \times 1}{100}$$

$$\Rightarrow r\% = \frac{y-x}{x} \times 100$$

Example-2:

An amount of money grows upto Rs 3000 in 3 years and upto Rs 4000 in 4 years on compound interest. What will be the rate percent?

Solution:

Principal + CI for 3 years = 3000 …… (1)

Principal + CI for 4 years= 4000 ……. (2)

Hence (2) – (1) =>CI for 4th year = 4000-3000= Rs 1000

Which is basically the simple interest upon 3000

Alternative Method:

Use the formula we have derived above:

$$R\% = \frac{y-x}{x} \times 100$$

$$R\% = \frac{4000-3000}{3000} \times 100 = \frac{100}{3}\%$$

If a certain sum becomes x times of itself in t years, the rate of compound interest will be equal to

$$r = 100[(x)^{\frac{1}{t}} - 1]$$

Example 4:

If a certain sum becomes 16 times in 2 years , what will be the rate of compound interest?

Solution:

Using the formula derived above:

$$r = 100[(x)^{\frac{1}{t}} - 1]$$

$$r = 100[(16)^{\frac{1}{2}} - 1] = 300\%$$

If the compound interest on a certain sum for 2 years is CI and simple interest for two years is SI ,then rate of interest per annum is

If the compound interest on a certain sum for 2 years is CI and simple interest for two years is SI ,then rate of interest per annum is

$$r\% = 2\frac{(CI - SI)}{SI} \times 100$$

Example 5:

If the compound interest on a certain sum for 2 years is 20rs and simple interest for two years is 10rs ,then what wil be the rate of interest per annum ?

Solution:

Using the formula derived above:

$$r\% = 2\frac{(CI - SI)}{SI} \times 100$$

$$r\% = \frac{2(20-10)}{10} \times 100 = 200\%$$

SOLVED EXERCISE

1. A bank offers 5% compound interest calculated on half-yearly basis. A customer deposits Rs. 1600 each on 1st January and 1st July of a year.
   At the end of the year, the amount he would have gained by way of interest is:
   (A) 123 (B) 122
   (C) 121 (D) 120
2. The compound interest on Rs. 30,000 at 7% per annum is Rs. 4347. The period (in years) is:
   (A) 2.5 B) 2
   (C) 3 D) 4
3. At what rate of compound interest per annum will a sum of Rs. 1200 become Rs. 1348.32 in 2 years?
   (A) 8 % (B) 9%
   (C) 6 % (D) 8.5 %
4. The difference between simple interest and compound on Rs. 1200 for one year at 10% per annum reckoned half-yearly is:
   (A) Rs. 3 (B) Rs. 4
   (C) Rs. 3.5 (D) Rs. 7.5
5. The least number of complete years in which a sum of money put out at 20% compound interest will be more than doubled is:
   (A) 4 (B) 5
   (C) 6 (D) 2.5
6. What will be the compound interest on a sum of Rs. 25,000 after 3 years at the rate of 12 p.c.p.a.?
   (A) Rs.10123.20 (B) Rs. 9000
   (C) Rs. 12000 (D) Rs. 10163.34
7. Simple interest on a certain sum of money for 3 years at 8% per annum is half the compound interest on Rs. 4000 for 2 years at 10% per annum. The sum placed on simple interest is:
   (A) Rs. 1650 (B) Rs. 2000
   (C) Rs. 1750 (D) Rs.1550

SOLVED EXERCISE EXPLANATION

1. Amount = Rs. [1600×(1+ 5/200)^2 + 1600 × (1+5/200)]
   = Rs. 3321
   So CI = Amount- Principal
   = Rs. 3321 – Rs. 3200 = Rs. 121
2. Amount = Rs. (30000 + 4347) = Rs. 34347,
   Let the time be n years then
   30000(1+7/100) ^n = 34347
   (107/100) ^n = 34347/30000
   So n= 2 year.
3. Let rate r % per annum
   1200× (1+r/100) ^ 2 = 1348.32
   (1+r/100) ^ 2 = 1348.32/1200
   1+r/100 = 106 / 100, r= 6 %
4. SI =Rs. (1200 ×10×1)/100= Rs. 120
   CI = Rs.[1200×(1+5/100) ^2 - 1200] = Rs.123
   So CI-SI = Rs. 3
5. P(1+20/100) ^n > 2P
   (6/5)^ n >2
   (6/5×6/5×6/5×6/5)>2
   so n = 4 years
6. Amount= Rs. 25000(1+12/100)^3= 35123.20
   So CI= Rs. (35123.20 - 25000) = Rs. 10123.20
7. C.I.= Rs. 4000(1+10/100)^2 – 40
   = Rs. 840
   Sum= Rs. (420 × 100)/(3×8) = Rs. 1750