UNDERSTANDING QUADRILATERALS

CONTENTS

- Polygons

POLYGONS

A simple closed figure made up of only line segments is called a polygon. If number of sides is n then number of angles is also n.

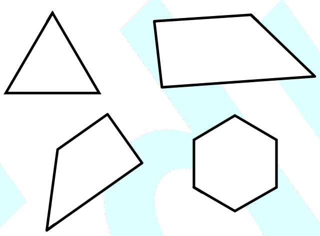
(n is natural no. ≥ 3)

Types : There are two types of polygon.

(1) Convex polygon   (2) Concave polygon

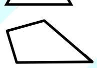
Convex polygon

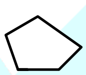
If each angle of a polygon is less than 180° then these are called convex polygon.

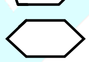


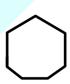
If n is number of sides, then

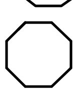
n = 3  triangle

n = 4  quadrilateral

n = 5  pentagon

n = 6  Hexagon

n = 7  Heptagon

n = 8  Octagon

for n = 9, n = 10 polygon are called nonagon & decagon respectively.

If sides of polygon are different with each other then interior (as well as exterior) angles are different and polygon is said to be irregular but if all sides are equal then polygon is said to be regular polygon, like equilateral triangle, square etc. In these, all angles are equal and value of each interior angle is $\frac{(n-2)\,180°}{n}$, n = number of sides.

also sum of all interior angles = (n – 2) 180°

❖ EXAMPLES ❖

Ex.1 Find the sum of all interior angles for a decagon.

Sol. $\because$ n = 10

$\therefore$ sum of angles = (n – 2) 180°

= (10 – 2) 180 = 8 × 180 = 1440°

Ex.2 If the sum of all angles of a polygon is 720° then find number of sides.

Sol. $\because$ sum of angles = (n – 2) 180°

$\Rightarrow$ 720° = (n – 2) 180°

$\Rightarrow n - 2 = \frac{720}{180} = 4 \Rightarrow n = 6$

Ex.3 If all sides are equal of a polygon of 15 sided then find value of each interior angle.

Sol. $\because$ n = 15

$\therefore$ each interior angle $= \frac{(n-2)180°}{n}$

$= \left(\frac{15-2}{15}\right)180° = 13 \times 12 = 156°$

Note :

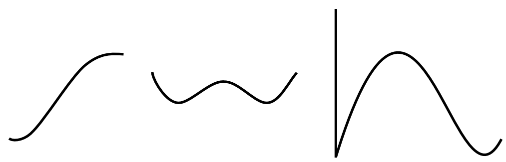
(1) Sum of all exterior angles is equal to 360°

(2) Value of each exterior angle of regular polygon is $\frac{360°}{n}$.

(3) Number of Diagonals in polygon = $\frac{n(n-3)}{2}$

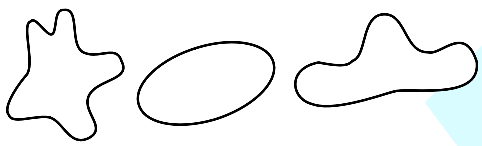
(4) Simple curve can be bound or not

Eg.



(5) Simple closed curve is not always circle

Eg.



Ex.4 Check whether 115° can an exterior angle of a regular polygon ?

Sol. ∵ each exterior angle = $\frac{360°}{n}$

$= \frac{360°}{115}$

≠ natural number

∴ 115° can not be an exterior angle of a regular polygon.

Ex.5 Find the number of diagonals if the sum of all interior angles is 900°.

Sol. ∵ sum of all interior angles

= (n – 2) 180°

900 = (n – 2) 180°

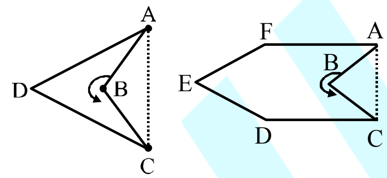
$\Rightarrow n - 2 = \frac{900}{180} = 5$

⇒ n = 7

Now no. of diagonals = $\frac{7(7-3)}{2} = \frac{7(4)}{2} = 14$

◈ **Concave polygon**

If value of one angle of a polygon is more than 180° then these polygon are called concave polygon. In these, one diagonal is in exterior of polygon.



∠B > 180° also dia. (AC) is in exterior

◈ **Angle sum property**

The sum of measure of all interior angles of a polygon is called angle sum property (a. s. p.)

Eg. A. S. P. for triangle, quadrilateral……are 180°, 360°…….

Ex.6 If two angles of a triangle are 40° & 58° then find the third angle.

Sol. ∵ The sum of all angles = 180° (A.S.P.)

40° + 58° + Third angle = 180°

∴ Third angle = 180° – 98° = 102°

Ex.7 If two angles of a hexagon are right angles & rest angles are same to each other then find the value of one of the other angles.

Sol. Let the other each angle = x°

∴ 90° + 90° + x + x + x + x = (n – 2) 180°

⇒ 180 + 4x = (6 – 2)180°

⇒ 4x = 720 – 180°

$\Rightarrow x = \frac{540}{4} = 135°$.

Ex.8 Find the maximum exterior and minimum interior angle of regular polygon.

Sol. ∵ Minimum number of sides in a regular polygon is 3 (equilateral Δ)

∴ each angle = x° (Let)

∴ 3x = 180 ⇒ x = 60°

∴ minimum value of interior angle = 60°

∴ maximum exterior angle = 120°

(by linear pair).

Ex.9 The angles of a quadrilateral are in ratio 1 : 3 : 7 : 9 find the measure of each angle.

Sol. Let angles are x°, 3x°, 7x°, 9x°

$\therefore$ x + 3x + 7x + 9x = 360° (A.S.P.)

$\Rightarrow$ 20x = 360

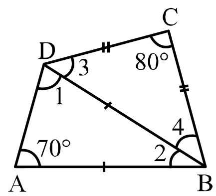
x = 18

$\therefore$ angles are 18, 18 × 3, 18 × 7, 18 × 9

= 18°, 54°, 126°, 162°

Verification 18° + 54° + 126° + 162° = 360°

Ex.10 Find the angles of quadrilateral ABCD, in given figure.



Sol. $\because$ AB = BD (in ΔADB)

$\therefore$ ∠1 = 70°

$\therefore$ ∠1 + 70° + ∠2 = 180° (A.S.P.)

$\Rightarrow$ 70 + 70 + ∠2 = 180°

$\Rightarrow$ ∠2 = 180° – 140° = 40°

also in ΔDCB

DC = CB

$\therefore$ ∠3 = ∠4 = y

$\therefore$ y + y + 80 = 180° (A.S.P.)

$\Rightarrow$ 2y = 180 – 80

$\Rightarrow$ $y = \frac{100}{2} = 50°$

$\therefore$ ∠3 = ∠4 = 50°

$\therefore$ ∠ABC = ∠2 + ∠4 = 40° + 50° = 90°

& ∠ADC = ∠1 + ∠3 = 70° + 50° = 120°

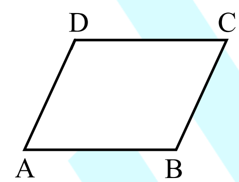
$\therefore$ angles are 70°, 90°, 80°, 120°.

◈ Types of quadrilateral :

(1) Parallelogram (2) Rhombus (3) Rectangle

(4) Square (5) Trapezium (6) Kite

(1) If opposite sides are equal and parallel then quadrilateral is called parallelogram ($\|^{gm}$)



Properties :

(i) Opposite sides are equal i.e. AB = CD, AD = BC

(ii) Opposite sides are parallel i.e. AB ∥ CD, AD ∥ BC

(iii) Opposite angles are equal i.e. ∠A = ∠C ; ∠B = ∠D

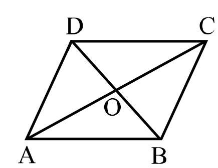
(iv) Sum of adjacent angles is 180° or adjacent angles are supplementary i.e. ∠A + ∠C = 180° or ∠A + ∠D = 180° etc.

(v) Length of both diagonals are different.

(vi) Diagonal bisect each other at same point.

(vii) One diagonal divides $\|^{gm}$ into two congruent triangles i.e. ΔABC ≅ ΔADC.

(2) A quadrilateral whose all sides are equal, is called rhombus. It is $\|^{gm}$ also $\because$ opposite sides are equal and parallel.



Properties :

(i) All sides are equal i.e. AB = BC = CD = DA

(ii) Opposite sides are parallel i.e. AB || CD, AD || BC

(iii) Opposite angles are equal i.e. ∠A = ∠C ; ∠B = ∠D

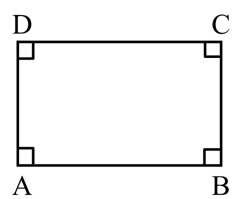
(iv) Sum of adjacent angles is 180° or adjacent angles are supplementary i.e. ∠A + ∠C = 180° or ∠A + ∠D = 180° etc.

(v) Length of both diagonals are different.

(vi) Diagonals bisect each other at right angle. i.e. AO = OC, OB = OD and BD ⊥ AC

(vii) A diagonal divides a rhombus into two congruent Δs.

(3) **Rectangle :** A rectangle is a $||^{gm}$ with all equal angles and value of each angle is 90°

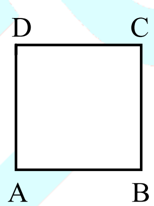


Properties :

(i) Diagonals are equal.

Other properties are same as parallelogram.

(4) **Square :** A square is a rectangle with all sides are equal or a square is a rhombus with all angles are equal (each 90°)

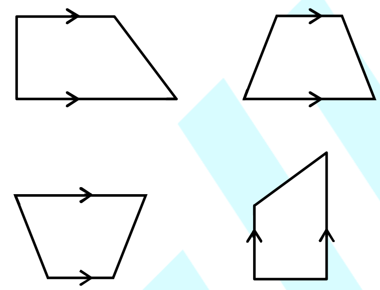


Properties :

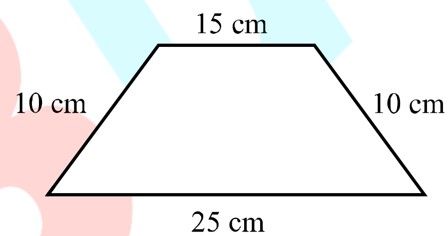
(i) Diagonals are of same length.

Other properties are same as rhombus.

(5) **Trapezium :** If opposite sides of one pair of quadrilateral are parallel & other two sides are non parallel then quadrilateral is called trapezium. The parallel sides are different in lengths.



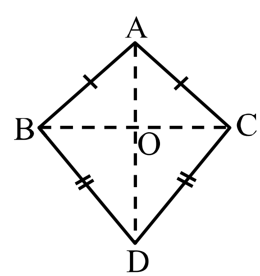
If non parallel sides are equal then it is called isosceles trapezium.



(6) **Kite :** This a is special type of a quadrilateral. In this adjacent sides are equal pair wise.

So there are exactly two distinct consecutive pairs of sides of equal length.

i.e. AB = AC and BD = DC



AD is longer diagonal & BC is smaller

AD bisects BC at right angle

i.e. BO = OC {not AO = OD}

also ∠BOD = ∠DOC = ∠AOC = ∠AOB = 90° or AD ⊥ BC.

IMPORTANT POINTS TO BE REMEMBERED

1. (i) A quadrilateral which has exactly one pair of parallel sides is called a trapezium.

(ii) A quadrilateral in which both pairs of opposite sides are parallel is called a parallelogram.

(iii) A parallelogram in which all the sides are equal is called a rhombus.

(iv) A parallelogram in which each angle is a right angle is called a rectangle.

(v) A parallelogram in which all the sides are equal and each angle is equal to a right angle is called a square.

(vi) A quadrilateral which has two pairs of equal adjacent sides but unequal opposite sides is called a kite.

2. A quadrilateral is a parallelogram if

(i) its opposite sides are equal, or

(ii) its opposite angles are equal, or

(iii) its diagonals bisect each other, or

(iv) it has one pair of opposite sides equal and parallel.

3. The diagonals of a rhombus bisect each other at right angles.

4. The diagonals of a rectangle are equal.

5. The diagonals of a square are equal and bisect each other at right angles.

6. One angle is more than 180° in concave polygon.

7. One diagonal is in exterior of concave polygon.

8. Both diagonal are in interior of quadrilateral.

9. Sum of interior angles = $(n - 2)\ 180°$

(n = number of sides).

10. Each interior angles of regular polygon

$= \frac{(n-2)180°}{n}$.

11. Each exterior angle of regular polygon = $\frac{360°}{n}$.

12. A line joining any two distinct points in quadrilateral is always in the quadrilateral.

13. A square, rectangle and rhombus are parallelograms.

14. A $\|^{gm}$ is a trapezium but a trapezium is not a $\|^{gm}$.

15. A rectangle or a rhombus is not necessarily a square.

16. A kite is not a $\|^{gm}$.