PLAYING WITH NUMBERS

CONTENTS

- Generalised form of 2 Digit and 3 Digit numbers
- Reversing the digits
- Pythagorean triplets
- Divisibility test

➢ GENERALISED FORM OF 2 DIGIT AND 3 DIGIT NUMBERS

(a) 2 digit number has the tens place and the units place

Eg. $45 = 4 \times 10 + 5$, $93 = 9 \times 10 + 3$

Eg. $ab = 10a + b$, $ba = 10b + a$

(b) 3 digit number has the hundreds place, the tens place and the units place.

Eg. $393 = 3 \times 100 + 9 \times 10 + 3$

Eg. $492 = 4 \times 100 + 9 \times 10 + 2$

Eg. $102 = 1 \times 100 + 0 \times 10 + 2$

Eg. $abc = 100\,a + 10\,b + c$

Eg. $cba = 100\,c + 10\,b + a$

Eg. The usual form of $10 \times 7 + 8$ and $10 \times 5 + 7$ are 78 and 57 respectively.

➢ REVERSING THE DIGITS

(a) **2 Digit Number :** If number is ab, $a \neq 0$ then reverse is ba. The difference of number & its reverse is divisible by 9.

Eg. Reverse of 23 or $2 \times 10 + 3$ is 32 or $3 \times 10 + 2$

also $32 - 23 = 9$ Its divisible by 9.

(b) **3 Digit Number :**

If number is abc or $100a + 10b + c$, $a \neq 0$ then the reverse is cba or $100c + 10b + a$

If $a > c$ then

$abc - cba = (100a + 10b + c) - (100c + 10b + a)$

$= 99 (a - c)$

If $c > a$ then

$cba - abc = (100c + 10b + a) - (100a + 10b + c)$

$= 99 (c - a)$

That means difference of a 3 digit number and its reverse number is divisible by 99.

* We can make more numbers from given no. abc like bca, acb, bac, cab etc.

also $abc + bca + cab = 111 (a + b + c)$

$= 37 \times 3 (a + b + c)$

$\therefore$ The number (abc + cab + bca) is divisible by 37, 3 and $a + b + c$.

eg. 927 :

$927 + 279 + 792 = 3 \times 37 (9 + 2 + 7)$

$= 3 \times 37 \times 18$

$1998 \div 3 = 666 = 18 \times 37$

$1998 \div 37 = 54 = 18 \times 3$

$1998 \div 18 = 111 = 3 \times 37$

Note : The first digit of a number can not be zero.

eg. 29 is a two digit number but 029 is not a 3 digit no.

➢ FIND THE DIGITS

Ex.1 Find the value of x.

$$\begin{array}{r} 3\ \ 1\ \ x \\ +\ 1\ \ x\ \ 3 \\ \hline 5\ \ 0\ \ 1 \end{array}$$

Sol. In ones column addition of x, 3 gives 1

$\therefore$ x may be 8

If x = 8 then we get a number whose ones digit is 1 & remaining 1 makes 2 + x in II column

$\therefore$ 2 + 8 = 10

So 0 is tens digit of result and remaining 1 makes 5 of sum of III column.

$\therefore$ x = 8

Ex.2 Find the value of x, y

$$\begin{array}{r} x \\ +\ x \\ +\ x \\ \hline y\ \ x \end{array}$$

Sol. If x = 5 then 5 + 5 + 5 = 15

$\therefore$ y = 1, x = 5.

Ex.3 Find the value of x, y

$$\begin{array}{r} 2\ \ 5\ \ x\ \ 4 \\ +\ y\ \ 5\ \ 2\ \ 8 \\ \hline 1\ \ 2\ \ 1\ \ 0\ \ 2 \end{array}$$

Sol. x = 7, y = 9

Ex.4 Find the value of x

$$\begin{array}{r} 2\ \ x\ \ 7 \\ 7\ \ 2\ \ x \\ +\ x\ \ 7\ \ 2 \\ \hline 1\ \ x\ \ x\ \ 2 \end{array}$$

Sol. x = 3

➢ PYTHAGOREAN TRIPLETS

If the square of a number is equal to sum of square other two numbers then these three numbers are called Pythagorean triplets.

eg. 3, 4, 5 here $5^2 = 3^2 + 4^2$

Other Pythagorean triplets are (5, 12, 13), (7, 24, 25), (6, 8, 10), (8, 15, 17) etc.

For any natural number m > 1,

we have $(2m)^2 + (m^2 - 1)^2 = (m^2 + 1)^2$.

So, 2m, $m^2 - 1$ and $m^2 + 1$ forms a Pythagorean triplet.

Ex.5 Write a Pythagorean triplet whose smallest member is 8.

Sol. We can get Pythagorean triplet by using general from 2m, $m^2 - 1$, $m^2 + 1$.

Let us first take $m^2 - 1 = 8$

So, $m^2 = 8 + 1 = 9$

which gives m = 3

Therefore, 2m = 6

and $m^2 + 1 = 10$

The triplet is thus 6, 8, 10. But 8 is not the smallest member of this.

So, let us try 2m = 8

then m = 4

We get $m^2 - 1 = 16 - 1 = 15$

and $m^2 + 1 = 16 + 1 = 17$

The triplet is 8, 15, 17 with 8 as the smallest member.

Ex.6 Find a Pythagorean triplet in which one member is 12.

Sol. If we take $m^2 - 1 = 12$

Then, $m^2 = 12 + 1 = 13$

Then the value of m will not be an integer.

So, we try to take $m^2 + 1 = 12$.

Again $m^2 = 11$ will not give an integer value for m.

So, let us try 2m = 12

then m = 6

Thus, $m^2 - 1 = 36 - 1 = 35$

and $m^2 + 1 = 36 + 1 = 37$

Therefore, the required triplet is 12, 35, 37.

Note : All Pythagorean triplets may not be obtained using this form. For example another triplet 5, 12, 13 also has 12 as a member.

➢ DIVISIBILITY TEST

No.	Divisibility Test
2	Unit digit should be 0 or even.
3	The sum of digits of no. should be divisible by 3.
4	The no. formed by last 2 digits of given no. should be divisible by 4.
5	Unit digit should be 0 or 5.
6	No. should be divisible by 2 & 3 both.
7	No. without ones – 2(ones) = no. should divisible by 7.
8	The number formed by last 3 digits of given no. should be divisible by 8.
9	Sum of digits of given no. should be divisible by 9.
11	The difference between sums of the digits at even & at odd places should be zero or multiple of 11.
13	No. without ones +4 (ones digit) = No. should be divisible by 13.
25	Last 2 digit of the number should be 00, 25, 50 or 75.

Ex.7 Check 119 and 329 is divisible by 7 or not.

Sol. (i) $11 - 2(9) = -7$, it is divisible by 7

$\therefore$ 119 is divisible by 7

(ii) $32 - 2(9) = 32 - 18 = 14$ is divisible by 7

$\therefore$ 329 is divisible by 7

Ex.8 Check 611 is divisible by 13 or not.

Sol. $61 + 4(1) = 61 + 4 = 65$

here 65 is divisible by 13

$\therefore$ 611 is divisible by 13