MENSURATION

CONTENTS

Area :

A figure made up of straight line segments is called a **rectilinear figure**.

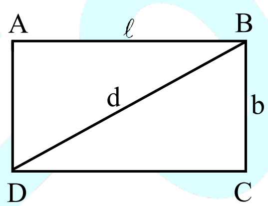
➤ AREA OF RECTANGLE AND SQUARE

◈ Rectangle :

Area = length × breadth or A = $\ell \times b$

Perimeter = 2 (length + breadth) or

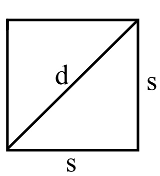
P = $2(\ell + b)$



◈ Square :

Area = $(\text{side})^2$ or A = s^2

Perimeter = 4 × side or P = 4s



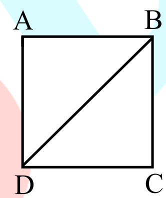
❖ EXAMPLES ❖

Ex.1 Show that area of a square = $\frac{1}{2} \times (\text{diagonal})^2$.

Find the area of a square whose diagonal = 2.5 cm.

Sol. In right triangle BCD

$(\text{diagonal})^2 = DC^2 + CB^2 = s^2 + s^2 = 2s^2$



But area of square = s^2

$\therefore$ $(\text{diagonal})^2 = 2 \times \text{area}$

or area = $\frac{1}{2} \times (\text{diagonal})^2$

If diagonal = 2.5 cm

$$\text{area} = \frac{1}{2} \times (2.5)^2 \text{ cm}^2 = \frac{6.25}{2} \text{ cm}^2 = 3.125 \text{ cm}^2.$$

Ex.2 The area of a square is 42.25 m^2. Find the side of the square. If tiles measuring 13 cm × 13 cm area paved on the square area. Find how many such tiles are used for paving it.

Sol. : The area of the square = 42.25 m^2

= 422500 cm^2

The side of the square = $\sqrt{\text{area}}$

$= \sqrt{422500}$ cm = 650 cm

The area of 1 tile = 13 cm × 13 cm = 169 cm^2

Number of tiles required

= 422500 ÷ 169 = 2500

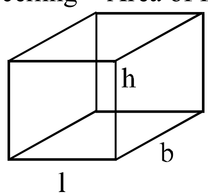
Ex.3 A room is 5 metres long. 4 metres broad and 3 metres high. Find the area of the four walls. Also find the area of the ceiling and the area of the floor. If it costs ₹ 0.30 to whitewash 1 dm^3 of wall, find the cost of whitewashing the four walls and the ceiling.

Sol.: Area of four walls

$= \ell h + bh + \ell h + bh = 2h(\ell+b)$

$= 6 \times 9\ m^2 = 54\ m^2$

Area of ceiling = Area of floor = 20 m^2



Since 1 m^2 = 100 dm^2,

$\therefore$ 54 m^2 = 5400 dm^2 and 20 m^2 = 2000 dm^2

Cost of whitewashing the four walls at the rate of ₹ 0.30 per dm^2

= ₹ (5400 × 0.30) = ₹ 1620

Cost of whitewashing the ceiling at the rate of ₹ 0.30 per dm^2

= ₹ (2000 × 0.30) = ₹ 600

Total cost of white washing

= ₹ 1620 + ₹ 600 = ₹ 2220

Ex.4 The length and breadth of a rectangular field is in the ratio 4 : 3. If the area is 3072 m^2, find the cost of fencing the field at the rate of ₹ 4 per metre.

Sol.: Let the length and breadth of the field be 4x and 3x metres respectively. The area of the field

$= 4x \times 3x = 12x^2 = 3072\ m^2$

Hence $x^2 = 3072 \div 12 = 256$

or $x = \sqrt{256} = 16$

Length = 4x = 64 m; Breadth = 3x = 48 m

Length of fencing = Perimeter of the field

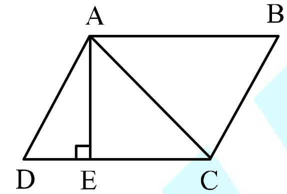
= 2 (64 + 48) m = 224 m

Cost of fencing at ₹ 4 per meter

= ₹ (224 × 4) = ₹ 896

➤ AREA OF QUADRILATERALS

◈ Area of a Parallelogram :



Consider parallelogram ABCD.

Let AC be a diagonal

In ΔADC and ΔCBA

AD = CB, CD = AB

AC is common

$\therefore$ $\Delta ADC \cong \Delta CBA$

$\therefore$ Area of parallelogram ABCD

= Area of ΔADC + Area of ΔABC

= 2 × Area of ΔADC

= 2 × (½ CD × AE) (where AE $\perp$ DC)

= DC × AE

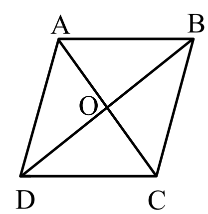
i.e. Area of parallelogram = base × height

◈ Area of a Rhombus

Since a rhombus is also a prallelogram, its area is given by

Area of rhombus = base × height

The area of a rhombus can also be found if the length of the diagonals are given. Let ABCD be a rhombus. We know that its diagonals AC and BD bisect each other at right angles.



Area of rhombus ABCD = area of ΔABD + area of ΔCBD

= ½ (BD × AO) + ½(BD × CO)

(since AO $\perp$ BD and CO $\perp$ BD)

= ½ BD (AO + CO) = ½ BD × AC

i.e. Area of rhombus = ½ × product of diagonals

◈ Area of a Trapezium :

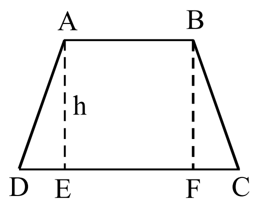
Let ABCD be a trapezium with AB || DC. Draw AE and BF perpendicular to DC.

Then AE = BF = height of trapezium = h

Area of trapezium ABCD = Area of ΔADE

+ Area of rectangle ABFE

+ Area of ΔBCF



$= \frac{1}{2} \times DE \times h + EF \times h + \frac{1}{2} FC \times h$

$= \frac{1}{2} h (DE + 2EF + FC)$

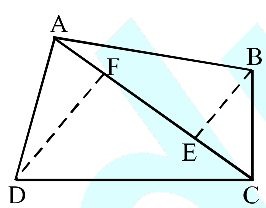
$= \frac{1}{2} h (DE + EF + FC + EF)$

$= \frac{1}{2} h (DC + AB)$ (since EF = AB)

i.e. Area of trapezium = ½ × (sum of parallel sides) × (distance between parallel sides)

◈ Area of a Quadrilateral :

Let ABCD be a quadrilateral, and AC be one of its diagonals. Draw perpendiculars BE and DF from B and D respectively to AC.



Area of quadrilateral ABCD

= Area of ΔABC + Area of ΔADC

$= \frac{1}{2} AC \times BE + \frac{1}{2} AC \times DF$

$= \frac{1}{2} AC (BE + DF)$

If AC = d, BE = h_1 and DF = h_2 then

Area of quadrilateral = $\frac{1}{2}d (h_1 + h_2)$

❖ EXAMPLES ❖

Ex.5 A rectangle and a parallelogram have the same area of 72 cm^2. The breadth of the rectangle is 8 cm. The height of the parallelogram is 9 cm. Find the base of the parallelogram and the length of the rectangle.

Sol. Area of rectangle = $\ell \times b = \ell \times 8 = 72$

$\therefore \quad \ell = 9$ cm

Area of parallelogram = base × height

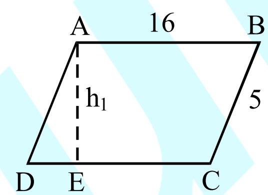
= base × 9 = 72

∴ Base = 8 cm

Ex.6 The area of a parallelogram is 64 cm^2. Its sides are 16 cm and 5 cm. Find the two heights of the parallelogram.

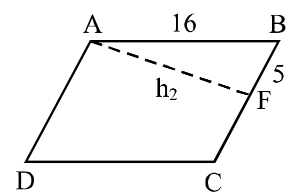
Sol. : (i) Area = base × height = $16 \times h_1 = 64$

$\therefore \quad h_1 = 4$cm



(ii) Area = base × height = $5 \times h_2 = 64$

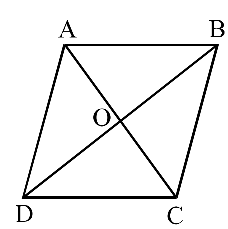
$\therefore \quad h_2 = 12.8$ cm



Ex.7 The diagonals of a rhombus measure 10 cm and 24 cm. Find its area. Also find the measure of its side.

Sol. : AC = 10 cm, BD = 24 cm

Area = $\frac{1}{2} (d_1 \times d_2) = \frac{1}{2} \times 10 \times 24\ cm^2 = 120\ cm^2$



In ΔABO, ∠AOB = 90º, AO = ½ AC = 5 cm,

BO = ½ BD = 12 cm.

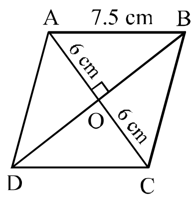
$\therefore AB^2 = AO^2 + OB^2 = 25 + 144 = 169 = 13 \times 13$

∴ AB = 13 cm

∴ Measure of ℓ side = 13 cm

Ex.8 In rhombus ABCD, AB = 7.5 cm, and AC = 12 cm. Find the area of the rhombus.

Sol. : In ΔABO, $\angle AOB = 90^\circ$, $AO = \frac{1}{2} AC = 6$ cm, $AB = 7.5$ cm



$\therefore$ $OB^2 = AB^2 - OA^2$

$= (7.5)^2 - 6^2 = 56.25 - 36 = 20.25$

$\therefore$ $OB = \sqrt{20.25} = 4.5$ cm

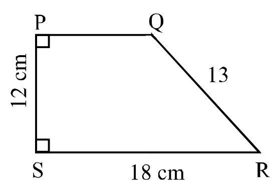
$\therefore$ $BD = 2 \times OB = 9$ cm

Area of rhombus $= \frac{1}{2} d_1 \times d_2$

$= \frac{1}{2} \times 9 \times 12 \text{ cm}^2 = 54 \text{cm}^2$

Ex.9 In the trapezium PQRS, $\angle P = \angle S = 90^\circ$, PQ = QR = 13 cm, PS = 12 cm and SR = 18 cm. Find the area of the trapezium.

Sol. : The parallel sides are PQ and SR, and the distance between them is PS, since $\angle P = \angle S = 90^\circ$



$\therefore$ Area $= \frac{1}{2} \times$ sum of parallel sides $\times$ heights

$= \frac{1}{2} \times (13 + 18) \times 12 \text{ cm}^2$

$= 186 \text{ cm}^2$

Ex.10 In trapezium ABCD, AB = AD = BC = 13 cm and CD = 23 cm. Find the area of the trapezium.

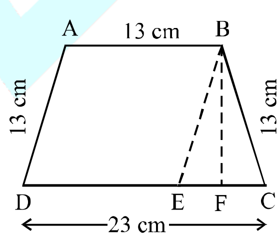
Sol. : From B draw BE || AD, and $BF \perp DC$

Since ABED is a parallelogram, DE, = 13 cm.

$\therefore$ EC = 23 cm – 13 cm = 10 cm

Also BE = 13 cm.

Therefore BEC is an isosceles triangle.



Since $BF \perp EC$, therefore F is the midpoint of EC

$\therefore$ $FC = \frac{1}{2} \times 10$ cm $= 5$ cm

In the right triangle BFC

$BF^2 = BC^2 - FC^2 = 13^2 - 5^2 = 144$

$\therefore$ BF = 12 cm

Area of trapezium = ½ sum of parallel sides × height

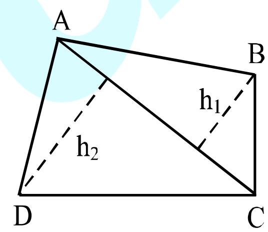
$= \frac{1}{2} (13 + 23) \times 12 \text{ cm}^2$

$= 216 \text{ cm}^2$

Note : We can also say : Area of ABCD = Area of $||^{gm}$ ABED + Area of ΔBCE (can be found by Hero's formula as all its sides are known).

Ex.11 In a quadrilateral ABCD, AC = 15 cm, The perpendiculars drawn from B and D respectively to AC measure 8.2 cm and 9.1 cm. Find the area of the quadrilateral.

Sol. :



Area of quadrilateral $= \frac{1}{2} d (h_1 + h_2)$

$= \frac{1}{2} \times 15 \times (8.2 + 9.1) \text{ cm}^2$

$= \frac{1}{2} \times 15 \times 17.3 \text{ cm}^2$

$= 129.75 \text{ cm}^2$

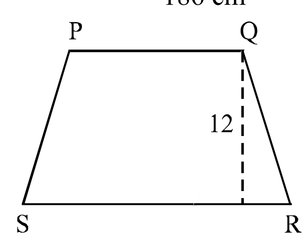
Ex.12 PQRS is a trapezium, in which SR || PQ, and SR is 5 cm longer than PQ. If the area of the trapezium is 186 cm^2 and the height is 12 cm, find the lengths of the parallel sides.

Sol. : Let PQ = x cm; then SR = (x + 5)

Area of PQRS

$= \frac{1}{2} \times 12 \times (x + x + 5) \text{ cm}^2$

$= 186 \text{ cm}^2$



$\therefore$ $6(2x + 5) = 186$

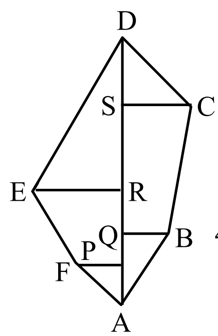
or $2x + 5 = 31$ $\therefore$ $x = 13$

$\therefore$ PQ = 13 cm, SR = 13 cm 5 cm = 18 cm

AREA OF IRREGULAR RECTILINEAR FIGURES

For field ABCDEF, to find its area, we proceeds as follows :

1. Select two farthest corners (A and D) such that the line joining them does not intersect any of the sides. Join the corners. The line joining them is called the **base line**. In this case the base line is AD.
2. From each corner draw perpendiculars FP, BQ, ER and CS to AD. These are called **offsets**.
3. Measure and record the following lengths: AP and PF, AQ and QB, AR and RE, AS and SC.
4. Record these measurements as shown.



	Metres	
	To D	
	250	
	180	50 to C
60 to E	100	
	60	30 to B
40 to F	30	
	From A	

The field has been divided into four right triangles and two trapezia. In the trapezia, the parallel sides are perpendicular to the base line.

The area of the field is the sum of the areas of the triangles and trapezia.

Area of $\Delta APF = \frac{1}{2} \times AP \times FP = \frac{1}{2} \times 30 \times 40\ m^2$

$= 600\ m^2$

Area of $\Delta AQB = \frac{1}{2} \times AQ \times QB = \frac{1}{2} \times 60 \times 30\ m^2$

$= 900\ m^2$

Area of trapezium PREF

$= \frac{1}{2} \times PR\ (PF + RE)$

$= \frac{1}{2} \times 70 \times 100\ m^2 = 3500\ m^2$

Area of trapezium BQSC

$= \frac{1}{2} \times QS\ (BQ + SC)$

$= \frac{1}{2} \times 120 \times 80\ m^2 = 4800\ m^2$

Area of $\Delta SCD = \frac{1}{2} \times SD \times SC$

$= \frac{1}{2} \times 70 \times 50\ m^2 = 1750\ m^2$

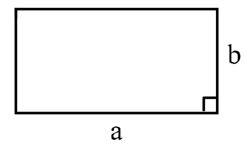
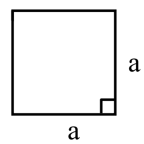
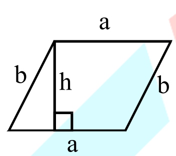
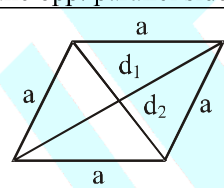
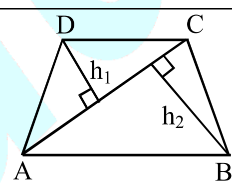
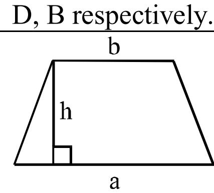
Area of $\Delta ERD = \frac{1}{2} \times RD \times ER$

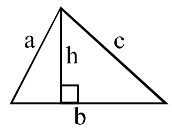
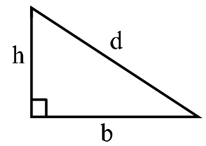
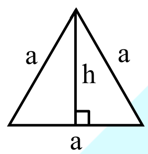
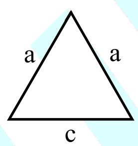
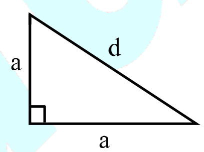
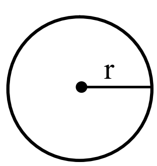
$= \frac{1}{2} \times 150 \times 60\ m^2 = 4500\ m^2$

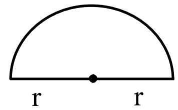
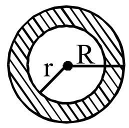
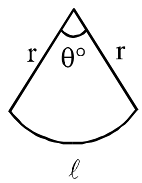
Total area $= (600 + 900 + 3500 + 4800 + 1750 + 4500)\ m^2$

$= 16050\ m^2$

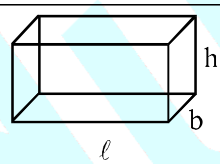
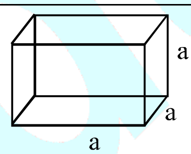
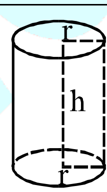
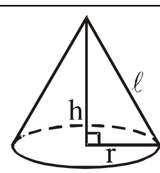
❖ Formulae to calculate area of some geometrical figures :

S. No.	Name	Figure	Perimeter in units of length	Area in square units
1.	Rectangle	 a = length, b = breadth	$2(a + b)$	ab
2.	Square	 a = side	$4a$	a^2 $\frac{1}{2}(\text{diagonal})^2$
3.	Parallelogram	 a = side b = side adjacent to a h = distance between the opp. parallel sides	$2(a + b)$	ah
4.	Rhombus	 a = side of rhombus $d_1 d_2$ are the two diagonals	$4a$	$\frac{1}{2} d_1 d_2$
5.	Quadrilateral	 AC is one of its diagonals and h_1, h_2 are the altitudes on AC from D, B respectively.	Sum of its four sides	$\frac{1}{2}(AC)(h_1 + h_2)$
6.	Trapezium	 a, b, are parallel sides and h is the distance between parallel sides	Sum of its four sides	$\frac{1}{2} h (a + b)$

S. No.	Name	Figure	Perimeter in units of length	Area in square units
7.	Triangle	 b is the base and h is the altitude a, b, c are three sides of Δ.	$a + b + c = 2s$ where s is the semi perimeter	$\frac{1}{2} b \times h$ or $\sqrt{s(s-a)(s-b)(s-c)}$
8.	Right triangle	 d(hypotenuse) $= \sqrt{b^2 + h^2}$	$b + h + d$	$\frac{1}{2} bh$
9.	Equilateral triangle	 a = side h = altitude = $\frac{\sqrt{3}}{2} a$	3a	(i) $\frac{1}{2} ah$ (ii) $\frac{\sqrt{3}}{4} a^2$
10.	Isosceles triangle	 c = unequal side a = equal side	$2a + c$	$\frac{c\sqrt{4a^2 - c^2}}{4}$
11.	Isosceles right triangle	 d(hypotenuse) $= a\sqrt{2}$, a = Each of equal sides, The angles are 90°, 45°, 45°.	$2a + d$	$\frac{1}{2} a^2$
12.	Circle	 r = radius of the circle $\pi = \frac{22}{7}$ or 3.1416	$2\pi r$	πr^2

S. No.	Name	Figure	Perimeter in units of length	Area in square units
13.	Semicircle	 r = radius of the circle	$\pi r + 2r$	$\frac{1}{2}\pi r^2$
14.	Ring (shaded region)	 R = outer radius r = inner radius	……..	$\pi(R^2 - r^2)$
15.	Sector of a circle	 θ° = central angle of The sector, r = radius of the sector ℓ = length of the arc	$\ell + 2r$ where $\ell = \frac{\theta}{360} \times 2\pi r$	$\frac{\theta}{360} \times \pi r^2$

◈ **Volume of some solid figures :**

S. No.	Nature of the solid	Shape of the solid	Lateral/curved surface area	Total surface area	Volume	Abbreviations used
1.	Cuboid		$2h(\ell + b)$	$2(\ell b + bh + \ell h)$	ℓbh	ℓ = length b = breadth h = height
2.	Cube		$4a^2$	$6a^2$	a^3	a = length of edge
3.	Right circular cylinder		$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$	r = radius of base h = height of the cylinder
4.	Right circular cone		$\pi r \ell$ where $\ell = \sqrt{r^2 + h^2}$	$\pi r(\ell + r)$	$\frac{1}{3}\pi r^2 h$	h = height r = radius ℓ = slant height

❖ EXAMPLES ❖

Ex.13 Find the volume and surface area of a cuboid of ℓ = 10 cm, b = 8 cm and h = 6 cm.

Sol.: $V = \ell \times b \times h = 10 \text{ cm} \times 8 \text{ cm} \times 6 \text{ cm} = 480 \text{ cm}^3$

Surface area = $2(\ell b + \ell h + bh)$

$= 2(10 \text{ cm} \times 8 \text{ cm} + 10 \text{ cm} \times 6 \text{ cm} + 8 \text{ cm} \times 6 \text{ cm})$

$= 2(80 + 60 + 48) \text{ cm}^2 = 376 \text{ cm}^2$

Ex.14 How many matchboxes of size 4 cm × 3 cm × 1.5 cm can be packed in a cardbord box of size 30 cm × 30 cm × 20 cm ?

Sol.: Volume of cardboard box = 30cm×30cm × 20cm $= 18000 \text{ cm}^3$

Volume of each matchbox

$= 4 \text{ cm} \times 3 \text{ cm} \times 1.5 \text{ cm}$

$= 18 \text{ cm}^3$

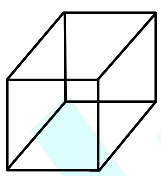
∴ Number of matchboxes that can fit in the cardboard box

$= 18000 \text{ cm}^3 \div 18 \text{ cm}^3 = 1000$

Ex.15 The dimensions of a cube are doubled. By how many times will its volume and surface area increase ?

Sol.: Let the side of the original cube be s

Then side of the new cube = 2s



(i) Volume of original cube = s × s × s

$= s^3$ cubic units

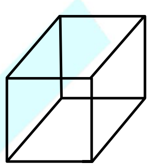
Volume of new cube = 2s × 2s × 2s

$= 8s^3$ cubic units

∴ Volume increases eight times if the side is doubled.

(ii) Surface area of original cube = $6s^2$

Surface area of new cube = $6(2s)^2 = 24s^2 = 4(6s^2)$



∴ Surface area increases four times.

Ex.16 The outer surface of a cube of edge 5m is painted. if the cost of painting is ₹ 1 per 100 cm^2, find the total cost of painting the cube.

Sol.: Surface area of cube = $6s^2 = 6 \times 5\text{m} \times 5\text{m} = 150\text{m}^2$

$= 150 \times 10000 \text{ cm}^2$

Cost of painting 100 cm^2 is ₹ 1.

∴ Cost of painting $150 \times 10000 \text{ cm}^2$ is

₹ $\frac{1}{100} \times 150 \times 10000$

= ₹ 15,000

Ex.17 A right circular cylinder has a height of 1 m and a radius of 35 cm. Find its volume, area of curved surface and total area.

Sol. h = 1m, r = 35 cm = 0.35 m

Volume = $\pi r^2 h = \frac{22}{7} \times 0.35 \times 0.35 \times 1 \text{ m}^3$

$= 0.385 \text{ m}^3$

Area of curved surface

$= 2\pi rh = 2 \times \frac{22}{7} \times 0.35 \times 1\text{m}^2 = 2.2 \text{ m}^2$

Total surface area = $2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 0.35 (1 + 0.35) \text{ m}^2$

$= \frac{2 \times 22 \times 0.35 \times 1.35}{7} \text{ m}^2 = 2.97 \text{ m}^2$

Ex.18 An open cylindrical tank is of radius 2.8m and height 3.5m. What is the capacity of the tank ?

Sol. Capacity = volume of cylinder

$= \pi r^2 h = \frac{22}{7} \times 2.8 \times 2.8 \times 3.5 \text{ m}^3$

$= 86.24 \text{ m}^3$

Ex.19 A metal pipe 154 cm long, has an outer radius equal to 5.5 cm and an inner radius of 4.5 cm. what is the volume of metal used to make the pipe ?

Sol. Outer volume = $\pi r^2 h = \frac{22}{7} \times (5.5)^2 \times 154 \text{ cm}^3$

Inner volume = $\frac{22}{7} \times (4.5)^2 \times 154 \text{ cm}^3$

∴ Volume of metal = outer volume – inner volume

$= \frac{22}{7} \times 154 \times (5.5)^2 - \frac{22}{7} \times 154 \times (4.5)^2$

$= \frac{22}{7} \times 154 [(5.5)^2 - (4.5)^2]$

$= \frac{22}{7} \times 154 (5.5 + 4.5)(5.5 - 4.5)$

$= \frac{22}{7} \times 154 \times 10 \times 1 = 4840 \text{ cm}^2$

Ex.20 A cylindrical roller is used to level a rectangular playground. The length of the roller is 3.5 m and its diameter is 2.8 m. if the roller rolls over 200 times to completely cover the playground, find the area of the playground.

Sol. : When the roller rolls over the ground once completely, It covers a ground area equal to its curved surface area.

Area of curved surface = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 1.4 \times 3.5 \text{ m}^2$$

$$\therefore \text{Area of ground} = \frac{200 \times 2 \times 22 \times 1.4 \times 3.5}{7} \text{m}^2$$

$$= 6160 \text{ m}^2$$

Ex.21 A cylindrical pipe has an outer diameter of 1.4m and an inner diameter of 1.12m. Its length is 10m. It has to be painted on the outer and inner surfaces as well as on the rims at the top and bottom. If the rate of painting is 0.01 per cm^2, find the cost of painting the pipe.

Sol. Outer surface area

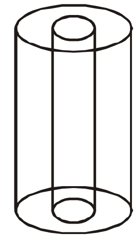
$$= 2\pi rh = 2 \times \frac{22}{7} \times 0.7 \times 10\text{m}^2$$

$$= 44\text{m}^2$$

Inner surface area

$$= 2\pi rh = 2 \times \frac{22}{7} \times 0.56 \times 10\text{m}^2$$

$$= 35.2\text{m}^2$$



Area of two rims = $2 \times \frac{22}{7} \times (0.7^2 - 0.56^2)$

$= 1.1088\text{m}^2$

$\therefore$ Total area to be painted

$= 44\text{m}^2 + 35.2\text{m}^2 + 1.1088\text{m}^2$

$= 80.3088 \text{ m}^2$

Rate of painting = ₹ 0.01 per cm^2

= ₹ 0.01×10000 per m^2

= ₹ 100 per m^2

$\therefore$ Total cost = ₹ 80.3088×100

= ₹ 8030.88

Ex.22 A rectangular piece of paper of width 20 cm and length 44 cm is rolled along its width to form a cylinder. What is the volume of the cylinder so formed ?

Sol. The length of the rectangle becomes the circumference of the base of the cylinder.

$\therefore 2\pi r = 44$, where r is the radius of the cylinder.

$\therefore$ $r = \frac{44 \times 7}{2 \times 22} = 7$ cm

The width of the rectangle becomes the height of the cylinder.

$\therefore$ Volume $= \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 20 \text{ cm}^3$

$= 3080 \text{ cm}^2$