LINEAR EQUATION IN ONE VARIABLE

CONTENTS

LINEAR EQUATION

Equation : A statement of equality which contains one or more unknown quantity or variable (literals) is called an equation. For example –

Ex.1 $3x + 7 = 12$, $\frac{5}{2}x - 9 = 1$, $x^2 + 1 = 5$ and $\frac{x}{3} + 5 = \frac{x}{2} - 3$ are equations in one variable x.

Ex.2 $2x + 3y = 15$, $7x - \frac{y}{3} = 3$ are equations in two variables x and y.

Linear Equation : An equation involving only linear polynomials is called a linear equation.

Ex.3 $3x - 2 = 7$, $\frac{3}{2}x + 9 = \frac{1}{2}$, $\frac{y}{3} + \frac{y-2}{4} = 5$ are linear equations in one variable, because the highest power of the variable in each equation is one whereas the equations $3x^2 - 2x + 1 = 0$, $y^2 - 1 = 8$ are not linear equations, because the highest power of the variable in each equation is not one.

SOLUTION OF A LINEAR EQUATION

Solution : A value of the variable which when substituted for the variable in an equation, makes L.H.S. = R.H.S. is said to satisfy the equation and is called a solution or a root of the equation.

◈ Rules for Solving Linear Equations in One Variable :

Rule-1 Same quantity (number) can be added to both sides of an equation without changing the equality.

Rule-2 Same quantity can be subtracted from both sides of an equation without changing the equality.

Rule-3 Both sides of an equation may be multiplied by the same non-zero number without changing the equality.

Rule-4 Both sides of an equation may be divided by the same non-zero number without changing the equality.

Solving Equations having Variable Terms on One Side and Number(s) on the Other Side :

❖ EXAMPLES ❖

Ex.1 Solve the equation : $\frac{x}{5} + 11 = \frac{1}{15}$ and check the result.

Sol. We have,

$\frac{x}{5} + 11 = \frac{1}{15} \Rightarrow \frac{x}{5} + 11 - 11 = \frac{1}{15} - 11$

[Subtracting 11 from both sides]

$\Rightarrow \frac{x}{5} = \frac{1}{15} - 11 \quad \Rightarrow \frac{x}{5} = \frac{1-165}{15}$

$\Rightarrow \frac{x}{5} = -\frac{164}{15} \quad \Rightarrow 5 \times \frac{x}{5} = 5 \times -\frac{164}{15}$

$\Rightarrow x = -\frac{164}{3}$

Thus, $x = -\frac{164}{3}$ is the solution of the given equation.

Check Substituting $x = \frac{-164}{3}$ in the given equation, we get

L.H.S. $= \frac{x}{5} + 11$

$= \frac{-164}{3} \times \frac{1}{5} + 11 = \frac{-164}{15} + 11$

$= \frac{164 + 165}{15} = \frac{1}{15}$ and,

R.H.S. $= \frac{1}{15}$

$\therefore$ L.H.S. = R.H.S. for $x = \frac{-164}{3}$

Hence, $x = \frac{-164}{3}$ is the solution of the given equation.

Ex.2 Solve : $\frac{1}{3}x - \frac{5}{2} = 6$

Sol. We have,

$\frac{1}{3}x - \frac{5}{2} = 6 \Rightarrow \frac{1}{3}x - \frac{5}{2} + \frac{5}{2} = 6 + \frac{5}{2}$

[Adding $\frac{5}{2}$ on both sides]

$\Rightarrow \frac{1}{3}x = 6 + \frac{5}{2} \quad \Rightarrow \frac{1}{3}x = \frac{12+5}{2}$

$\Rightarrow \frac{1}{3}x = \frac{17}{2} \quad \Rightarrow 3 \times \frac{1}{3}x = 3 \times \frac{17}{2}$

[Multiplying both sides by 3]

$\Rightarrow x = \frac{51}{2}$

Thus, $x = \frac{51}{2}$ is the solution of the given equation.

Check Substituting $x = \frac{51}{2}$ in the given equation, we get

L.H.S. $= \frac{1}{3}x - \frac{5}{2} = \frac{1}{3} \times \frac{51}{2} - \frac{5}{2}$

$= \frac{17}{2} - \frac{5}{2} = \frac{17-5}{2} = \frac{12}{2} = 6$

and, R.H.S. = 6

$\therefore$ L.H.S. = R.H.S. for $x = \frac{51}{2}$

Hence, $x = \frac{51}{2}$ is the solution of the given equation.

Ex.3 Solve : $\frac{x}{2} - \frac{x}{3} = 8$

Sol. We have, $\frac{x}{2} - \frac{x}{3} = 8$

LCM of denominators 2 and 3 on L.H.S. is 6. Multiplying both sides by 6, we get

$\Rightarrow 3x - 2x = 6 \times 8 \quad \Rightarrow x = 48$

Check Substituting x = 48 in the given equation, we get

L.H.S. $= \frac{x}{2} - \frac{x}{3} = \frac{48}{2} - \frac{48}{3} = 24 - 16 = 8$ and,

R.H.S. = 8

$\therefore$ L.H.S. = R.H.S. for x = 48

Hence, x = 48 is the solution of the given equation.

Ex.4 Solve : $\frac{x}{2} + \frac{x}{3} - \frac{x}{4} = 7$

Sol. We have, $\frac{x}{2} + \frac{x}{3} - \frac{x}{4} = 7$

LCM of denominators 2, 3, 4 on L.H.S. is 12. Multiplying both sides by 12, we get

$6x + 4x - 3x = 7 \times 12$

$\Rightarrow 7x = 7 \times 12 \quad \Rightarrow 7x = 84$

$\Rightarrow \frac{7x}{7} = \frac{84}{7}$ [Dividing both sides by 7]

$\Rightarrow x = 12$

Check Substituting x = 12 in the given equation, we get

L.H.S. $= \frac{12}{2} + \frac{12}{3} - \frac{12}{4} = 6 + 4 - 3 = 7$

and, R.H.S.= 7

$\therefore$ L.H.S. = R.H.S. for x = 12.

Hence, x = 12 is the solution of the given equation.

Ex.5 Solve : $\frac{y-1}{3} - \frac{y-2}{4} = 1$

Sol. We have, $\frac{y-1}{3} - \frac{y-2}{4} = 1$

LCM of denominators 3 and 4 on L.H.S. is 12.
Multiplying both sides by 12, we get

$12 \times \left(\frac{y-1}{3}\right) - 12 \times \left(\frac{y-2}{4}\right) = 12 \times 1$

$\Rightarrow$ 4 (y – 1) – 3(y – 2) = 12

$\Rightarrow$ 4y – 4 – 3y + 6 = 12

$\Rightarrow$ 4y – 3y – 4 + 6 =12

$\Rightarrow$ y + 2 = 12

$\Rightarrow$ y + 2 – 2 = 12 – 2 [Subtracting 2 from both sides]

$\Rightarrow$ y = 10

Thus, y = 10 is the solution of the given equation.

Check Substituting y = 10 in the given equation, we get

L.H.S. = $\frac{10-1}{3} - \frac{10-2}{3} = \frac{9}{3} - \frac{8}{4} = 3 - 2 = 1$

and, R.H.S. = 1

$\therefore$ L.H.S. = R.H.S. for y = 10.

Hence, y = 10 is the solution of the given equation.

Transposition Method for Solving Linear Equations in One Variable

The transposition method involves the following steps:

Step-I Obtain the linear equation.

Step-II Identify the variable (unknown quantity) and constants(numerals).

Step-III Simplify the L.H.S. and R.H.S. to their simplest forms by removing brackets.

Step-IV Transpose all terms containing variable on L.H.S. and constant terms on R.H.S. Note that the sign of the terms will change in shifting them from L.H.S. to R.H.S. and vice-versa.

Step-V Simplify L.H.S. and R.H.S. in the simplest form so that each side contains just one term.

Step-VI Solve the equation obtained in step V by dividing both sides by the coefficient of the variable on L.H.S.

❖ EXAMPLES ❖

Ex.6 Solve : $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$

Sol. We have, $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$

The denominators on two sides are 2, 5, 3 and 4. Their LCM is 60. Multiplying both sides of the given equation by 60, we get

$$60 \times \left(\frac{x}{2} - \frac{1}{5}\right) = 60 \left(\frac{x}{3} + \frac{1}{4}\right)$$

$\Rightarrow 60 \times \frac{x}{2} - 60 \times \frac{1}{5} = 60 \times \frac{x}{3} + 60 \times \frac{1}{4}$

$\Rightarrow$ 30x – 12 = 20x + 15

$\Rightarrow$ 30x – 20x = 15 + 12 [On transposing 20x to LHS and –12 to RHS]

$\Rightarrow 10x = 27 \Rightarrow x = \frac{27}{10}$

Hence, x = $\frac{27}{10}$ is the solution of the given equation.

Check Substituting x = $\frac{27}{10}$ in the given equation, we get

L.H.S. = $\frac{x}{2} - \frac{1}{5} = \frac{27}{10} \times \frac{1}{2} - \frac{1}{5} = \frac{27}{10} - \frac{1}{5}$

$= \frac{27 - 1\times4}{20} = \frac{27-4}{20} = \frac{23}{20}$

and,

R.H.S. = $\frac{x}{3} + \frac{1}{4} = \frac{27}{10} \times \frac{1}{3} + \frac{1}{4}$

$= \frac{9}{10} + \frac{1}{4} = \frac{9\times2+1\times5}{20} = \frac{18+5}{20} = \frac{23}{20}$

Thus, for x = $\frac{27}{10}$, we have L.H.S. = R.H.S.

Ex.7 Solve : $x + 7 - \frac{8x}{3} = \frac{17}{6} - \frac{5x}{8}$

Sol. We have, $x + 7 - \frac{8x}{3} = \frac{17}{6} - \frac{5x}{8}$

The denominators on two sides are 3, 6 and 8. Their LCM is 24.

Multiplying both sides of the given equation 24, we get

$$24\left(x + 7 - \frac{8x}{3}\right) = 24\left(\frac{17}{6} - \frac{5x}{8}\right)$$

$\Rightarrow$ $24x + 24 \times 7 - 24 \times \frac{8x}{3}$

$$= 24 \times \frac{17}{6} - 24 \times \frac{5x}{8}$$

$\Rightarrow$ $24x + 168 - 64x = 68 - 15x$

$\Rightarrow$ $168 - 40x = 68 - 15x$

$\Rightarrow$ $-40x + 15x = 68 - 168$

[Transposing –15x to LHS and 168 to RHS]

$\Rightarrow$ $-25x = -100$

$\Rightarrow$ $25x = 100$

$\Rightarrow$ $x = \frac{100}{25}$ [Dividing both sides by 25]

$\Rightarrow$ $x = 4$

Thus, x = 4 is the solution of the given equation.

Check Substituting x = 4 in the given equation, we get

L.H.S. $= x + 7 - \frac{8x}{3} = 4 + 7 - \frac{8 \times 4}{3}$

$$= 11 - \frac{32}{3} = \frac{33 - 32}{3} = \frac{1}{3}$$

and, R.H.S. $= \frac{17}{6} - \frac{5x}{8} = \frac{17}{6} - \frac{5 \times 4}{8} = \frac{17}{6} - \frac{5}{2}$

$$= \frac{17 - 15}{6} = \frac{2}{6} = \frac{1}{3}$$

Thus, for x = 4, we have L.H.S. = R.H.S.

Ex.8 Solve : $\frac{3t - 2}{4} - \frac{2t + 3}{3} = \frac{2}{3} - t$

Sol. We have, $\frac{3t - 2}{4} - \frac{2t + 3}{3} = \frac{2}{3} - t$

The denominators on two sides are 4, 3 and 3. Their LCM is 12.

Multiplying both sides of the given equation by 12, we get

$$12\left(\frac{3t - 2}{4}\right) - 12\left(\frac{2t + 3}{3}\right) = 12\left(\frac{2}{3} - t\right)$$

$\Rightarrow$ $3(3t - 2) - 4(2t + 3) = 12\left(\frac{2}{3} - t\right)$

$\Rightarrow$ $9t - 6 - 8t - 12 = 12 \times \frac{2}{3} - 12t$

$\Rightarrow$ $9t - 6 - 8t - 12 = 8 - 12t$

$\Rightarrow$ $t - 18 = 8 - 12t$

$\Rightarrow$ $t + 12t = 8 + 18$ [Transposing –12t to LHS and – 18 to RHS]

$\Rightarrow$ $13t = 26$

$\Rightarrow$ $t = \frac{26}{13}$ [Dividing both sides by 13]

$\Rightarrow$ $t = 2$

Check Substituting t = 2 on both sides of the given equation, we get

L.H.S. $= \frac{3t - 2}{4} - \frac{2t + 3}{3}$

$$= \frac{3 \times 2 - 2}{4} - \frac{2 \times 2 + 3}{3} = \frac{6 - 2}{4} - \frac{4 + 3}{3}$$

$$= \frac{4}{4} - \frac{7}{3} = 1 - \frac{7}{3} = \frac{3 - 7}{3} = \frac{-4}{3}$$

and,

R.H.S. $= \frac{2}{3} - t = \frac{2}{3} - 2 = \frac{2 - 6}{4} = \frac{-4}{3}$

Thus, for t = 2, we have L.H.S. = R.H.S.

Ex.9 Solve : $\frac{x+2}{6} - \left(\frac{11-x}{3} - \frac{1}{4}\right) = \frac{3x-4}{12}$

Sol. We have, $\frac{x+2}{6} - \left(\frac{11-x}{3} - \frac{1}{4}\right) = \frac{3x-4}{12}$

The denominators on two sides of the given equation are 6, 3, 4 and 12. Their LCM is 24.

Multiplying both sides of the given equation by 24, we get

$$24\left(\frac{x+2}{6}\right) - 24\left(\frac{11-x}{3} - \frac{1}{4}\right) = 24\left(\frac{3x-4}{12}\right)$$

$\Rightarrow$ $4(x+2) - 24\left(\frac{11-x}{3}\right) + 24 \times \frac{1}{4} = 2(3x-4)$

$\Rightarrow$ $4(x + 2) - 8(11 - x) + 6 = 2(3x - 4)$

$\Rightarrow$ $4x + 8 - 88 + 8x + 6 = 6x - 8$

$\Rightarrow$ $12x - 74 = 6x - 8$

$\Rightarrow$ $12x - 6x = 74 - 8$ [Transposing 6x to LHS and – 74 to RHS]

$\Rightarrow$ $6x = 66$

$\Rightarrow$ $x = \frac{66}{6}$ [Dividing both sides by 6]

$\Rightarrow$ $x = 11$

Check Substituting x = 11 on both sides of the given equation, we get

L.H.S. = $\frac{x+2}{6} - \left(\frac{11-x}{3} - \frac{1}{4}\right)$

$= \frac{11+2}{6} - \left(\frac{11-11}{3} - \frac{1}{4}\right) = \frac{13}{6} - \left(0 - \frac{1}{4}\right)$

$= \frac{13}{6} + \frac{1}{4} = \frac{26+3}{12} = \frac{29}{12}$

and, R.H.S. = $\frac{3x-4}{12} = \frac{3\times 11-4}{12} = \frac{33-4}{12} = \frac{29}{12}$

Thus, for x = 11, we have L.H.S. = R.H.S.

Ex.10 Solve : $x - \frac{2x+8}{3} = \frac{1}{4}\left(x - \frac{2-x}{6}\right) - 3$

Sol. We have,

$x - \frac{2x+8}{3} = \frac{1}{4}\left(x - \frac{2-x}{6}\right) - 3$

$\Rightarrow$ $x - \frac{2x+8}{3} = \frac{x}{4} - \frac{2-x}{24} - 3$

The denominators on the two sides of this equation are 3, 4 and 24. Their LCM is 24.

Multiplying both sides of this equation by 24, we get

$24x - 24\left(\frac{2x+8}{3}\right)$

$= 24 \times \frac{x}{4} - 24\left(\frac{2-x}{24}\right) - 3 \times 24$

$\Rightarrow$ $24x - 8(2x + 8) = 6x - (2 - x) - 72$

$\Rightarrow$ $24x - 16x - 64 = 6x - 2 + x - 72$

$\Rightarrow$ $8x - 64 = 7x - 74$

$\Rightarrow$ $8x - 7x = 64 - 74$ [Transposing 7x to LHS and – 64 to RHS]

$\Rightarrow$ $x = -10$

Thus, x = – 10 is the solution of the given equation.

Check Putting x ≡ 10 in LHS $= -10 - \frac{2\times(-10)+8}{3}$

$= -10 - \frac{-20+8}{3} = -10 - \left(\frac{-12}{3}\right) = -10 + 4 = -6$

and,

R.H.S. = $\frac{1}{4}\left(x - \frac{2-x}{6}\right) - 3 = \frac{1}{4}\left(-10 - \frac{2+10}{6}\right) - 3$

$= \frac{1}{4}(-10-2) - 3 = -3 - 3 = -6$

Thus, L.H.S. = R.H.S. for x = – 10.

Ex.11 Solve : 0.16 (5x – 2) = 0.4x + 7

Sol. We have,

$0.16(5x - 2) = 0.4x + 7$

$\Rightarrow 0.8x - 0.32 = 0.4x + 7$ [Expanding the bracket on LHS]

$\Rightarrow 0.8x - 0.4x = 0.32 + 7$ [Transposing 0.4x to LHS and –0.32 to RHS]

$\Rightarrow 0.4x = 7.32 \Rightarrow \frac{0.4x}{0.4} = \frac{7.32}{0.4}$

$\Rightarrow x = \frac{732}{40} \Rightarrow x = \frac{183}{10} = 18.3$

Hence, x = 18.3 is the solution of the given equation.

Ex.12 Solve : $\frac{2}{5x} - \frac{5}{3x} = \frac{1}{15}$

Sol. We have, $\frac{2}{5x} - \frac{5}{3x} = \frac{1}{15}$

Multiplying both sides by 15x, the LCM of 5x and 3x, we get

$15x \times \frac{2}{5x} - 15x \times \frac{5}{3x} = 15x \times \frac{1}{15}$

$\Rightarrow 6 - 25 = x \Rightarrow -19 = x \Rightarrow x = -19$

Hence, x = –19 is the solution of the given equation.

Ex.13 Solve : $\frac{17-3x}{5} - \frac{4x+2}{3} = 5 - 6x + \frac{7x+14}{3}$

Sol. Multiplying both sides by 15 i.e. the LCM of 5 and 3, we get

$3(17 - 3x) - 5(4x + 2)$

$= 15(5 - 6x) + 5(7x + 14)$

$\Rightarrow 51 - 9x - 20x - 10 = 75 - 90x + 35x + 70$

$\Rightarrow 41 - 29x = 145 - 55x$

$\Rightarrow -29x + 55x = 145 - 41$

$\Rightarrow 26x = 104 \Rightarrow \frac{26x}{26} = \frac{104}{26} \Rightarrow x = 4$

Thus, x = 4 is the solution of the given equation.

Ex.14 Solve : $\frac{x+2}{3} - \frac{x+1}{5} = \frac{x+3}{4} - 1$

Sol. Multiplying both sides by 60 i.e. the LCM of 3, 5, and 4, we get

$20(x + 2) - 12(x + 1) = 15(x - 3) - 1 \times 60$

$\Rightarrow 20x + 40 - 12x + 12 = 15x - 45 - 60$

$\Rightarrow 8x + 28 = 15x - 105 \Rightarrow 8x - 15x = 105 - 28$

$\Rightarrow -7x = -133$

$\Rightarrow \frac{-7x}{-7} = \frac{-133}{-7}$ [Dividing both sides by –7]

$\Rightarrow x = \frac{133}{7} = 19$

Thus, x = 19 is the solution of the given equation.

Ex.15 Solve : $(2x+3)^2 + (2x-3)^2 = (8x+6)(x-1) + 22$

Sol. We have,

$(2x + 3)^2 + (2x - 3)^2 = (8x + 6)(x - 1) + 22$

$\Rightarrow 2\{(2x)^2 + 3^2\}$

$= x(8x + 6) - (8x + 6) + 22$ [Using: $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$ on LHS]

$\Rightarrow 2(4x^2 + 9) = 8x^2 + 6x - 8x - 6 + 22$

$\Rightarrow 8x^2 + 18 = 8x^2 - 2x + 16$

$\Rightarrow 8x^2 - 8x^2 + 2x = 16 - 18$

$\Rightarrow 2x = -2$

$\Rightarrow x = -1$

Hence, x = – 1 is the solution of the given equation.

Cross-Multiplication Method for Solving Equations of the form :

$\frac{ax+b}{cx+d} = \frac{m}{n}$

$\Rightarrow n(ax + b) = m(cx + d)$

❖ EXAMPLES ❖

Ex.16 Solve : $\frac{2x+1}{3x-2}=\frac{9}{10}$

Sol. We have, $\frac{2x+1}{3x-2}=\frac{9}{10}$

$\Rightarrow$ 10 × (2x + 1) = 9 × (3x – 2)

[By cross-multiplication]

$\Rightarrow$ 20x + 10 = 27x – 18

$\Rightarrow$ 20x – 27x = – 18 – 10

[Using transposition]

$\Rightarrow$ – 7x = – 28

$\Rightarrow \frac{-7x}{-7}=\frac{-28}{-7}$ [Dividing both sides by –7]

$\Rightarrow$ x = 4

Hence, x = 4 is the solution of the given equation.

Ex.17 Solve : $\frac{3x+5}{2x+7}=4$

Sol. We have, = $\frac{3x+5}{2x+7}=4$

$\Rightarrow \frac{3x+5}{2x+7}=\frac{1}{4}$

$\Rightarrow$ 1 × (3x + 5) = 4 × (2x + 7)

[By cross-multiplication]

$\Rightarrow$ 3x + 5 = 8x + 28

$\Rightarrow$ 3x – 8x = 28 – 5 [Using transposition]

$\Rightarrow$ – 5x = 23

$\Rightarrow \frac{-5x}{-5}=\frac{23}{-5} \Rightarrow x=-\frac{23}{5}$

Hence, $x=-\frac{23}{5}$ is the solution of the given equation.

Ex.18 Solve : $\frac{17(2-x)-5(x+12)}{1-7x}=8$

Sol. We have, $\frac{17(2-x)-5(x+12)}{1-7x}=8$

$\Rightarrow \frac{34-17x-5x+60}{1-7x}=\frac{8}{1}$

$\Rightarrow \frac{-22x-26}{1-7x}=\frac{8}{1}$

$\Rightarrow$ 1 × (– 22x – 26) = 8 × (1 – 7x)

[By cross-multiplication]

$\Rightarrow$ – 22x – 26 = 8 – 56x

$\Rightarrow$ – 22x + 56x = 8 + 26

$\Rightarrow$ 34x = 34 $\Rightarrow \frac{34x}{34}=\frac{34}{34}$

Hence, x = 1 is the solution of the given equation.

Ex.19 Solve : $\frac{x+b}{a-b}=\frac{x-b}{a+b}$

Sol. We have, $\frac{x+b}{a-b}=\frac{x-b}{a+b}$

$\Rightarrow$ (x + b) × (a + b) = (x – b) × (a – b)

[By cross-multiplication]

$\Rightarrow$ x (a + b) + b(a + b) = x(a – b) – b(a – b)

$\Rightarrow ax+bx+ba+b^2=ax-bx-ba-b^2$

$\Rightarrow ax+bx-ax+bx=-bx+b^2-ba-b^2$

$\Rightarrow$ 2bx = – 2ba $\Rightarrow \frac{2bx}{2b}=-\frac{2ab}{2b}$

$\Rightarrow$ x = – a

Hence, x = – a is the solution of the given equation.

Ex.20 Solve : $\frac{(4+x)(5-x)}{(2+x)(7-x)}=1$

Sol. We have, $\frac{(4+x)(5-x)}{(2+x)(7-x)}=1$

$\Rightarrow \frac{20-4x+5x-x^2}{14-2x+7x-x^2}=1 \Rightarrow \frac{20+x-x^2}{14+5x-x^2}=1$

$\Rightarrow 20+x-x^2=14+5x-x^2$

[By cross-multiplication]

$\Rightarrow x-x^2=-5x+x^2=14-20$

$\Rightarrow -4x=-6 \Rightarrow \frac{-4x}{-4}=\frac{-6}{-4} \Rightarrow x=\frac{3}{2}$

Hence, $x=\frac{3}{2}$ is the solution of the given equation.

Ex.21 Solve : $\frac{1}{x+1} + \frac{1}{x+2} = \frac{2}{x+10}$

Sol. We have, $\frac{1}{x+1} + \frac{1}{x+2} = \frac{2}{x+10}$

Multiplying both sides by (x + 1)(x + 2)(x + 10) i.e., the LCM of x + 1, x + 2 and x + 10, we get

$$\frac{(x+1)(x+2)(x+10)}{x+1} + \frac{(x+1)(x+2)(x+10)}{x+2} = \frac{2(x+1)(x+2)(x+10)}{x+10}$$

$\Rightarrow$ $(x + 2)(x + 10) = (x + 1)(x + 10)$
$= 2(x + 1)(x + 2)$

$\Rightarrow$ $x^2 + 2x + 10x + 20 + x^2 + 10x + x + 10$
$= 2(x^2 + x + 2x + 2)$

$\Rightarrow$ $2x^2 + 23x + 30 = 2(x^2 + 3x + 2)$

$\Rightarrow$ $2x^2 + 23x + 30 = 2x^2 + 6x + 4$

$\Rightarrow$ $2x^2 + 23x - 2x^2 + 6x = 4 - 30$

$\Rightarrow$ $17x = -26$ $\Rightarrow$ $x = -\frac{26}{17}$

Hence, $x = -\frac{26}{17}$ is the solution of the given equation.

Ex.22 Solve : $\frac{6x^2+13x-4}{2x+5} = \frac{12x^2+5x-2}{4x+3}$

Sol. We have, $\frac{6x^2+13x-4}{2x+5} = \frac{12x^2+5x-2}{4x+3}$

$\Rightarrow$ $(6x^2 + 13x - 4)(4x+3) = (12x^2 + 5x - 2)(2x + 5)$

[By cross-multiplication]

$\Rightarrow$ $(6x^2 + 13x - 4) \times 4x + (6x^2 + 13x - 4) \times 3$
$= (12x^2 + 5x - 2) \times 2x + (12x^2 + 5x - 2) \times 5$

$\Rightarrow$ $24x^3 + 52x^2 - 16x + 18x^2 + 39x^2 - 12$
$= 24x^3 + 10x^2 - 4x + 60x^2 + 25x - 10$

$\Rightarrow$ $24x^3 + 70x^2 + 23x - 12$
$= 24x^3 + 70x^2 + 12x - 10$

$\Rightarrow$ $24x^3 + 70x^2 + 23x - 24x^3 - 70x^2 - 21x$
$= -10 + 12$

$\Rightarrow$ $2x = 2$ $\Rightarrow$ $x = 1$

Hence, x = 1 is the solution of the given equation.

Ex.23 Solve :

$$\frac{4x+17}{18} - \frac{13x-2}{17x-32} + \frac{x}{3} = \frac{7x}{12} - \frac{x+16}{36}$$

Sol. We have,

$$\frac{4x+17}{18} - \frac{13x-2}{17x-32} + \frac{x}{3} = \frac{7x}{12} - \frac{x+16}{36}$$

$$\Rightarrow \frac{4x+17}{18} - \frac{7x}{12} + \frac{x+16}{36} + \frac{x}{3} = \frac{13x-2}{17x-32}$$

Multiplying both sides by 36 i.e., the LCM of 18, 12, 36 and 3, we get

$$36 \times \frac{4x+17}{18} - 36 \times \frac{7x}{12} + 36 \times \frac{x+16}{36} + 36 \times \frac{x}{3}$$
$$= 36 \times \left(\frac{13x-2}{17x-32}\right)$$

$\Rightarrow$ $2(4x + 17) - 3 \times 7x + x + 16 + 12x$
$= 36 \times \left(\frac{13x-2}{17x-32}\right)$

$\Rightarrow$ $8x + 34 - 21x + x + 16 + 12x$
$= 36 \times \left(\frac{13x-2}{17x-32}\right)$

$\Rightarrow$ $50 = 36 \times \left(\frac{13x-2}{17x-32}\right)$

[By cross-multiplication]

$\Rightarrow$ $50 \times (17x - 32) = 36(13x - 2)$

$\Rightarrow$ $850x - 1600 = 468x - 72$

$\Rightarrow$ $850x - 468x = 1600 - 72$

$\Rightarrow$ $382x - 1528$

$\Rightarrow$ $x = \frac{1528}{382} = 4$

Hence, x = 4 is the solution of the given equation.

Applications of Linear Equations to Practical Problems

The following steps should be followed to solve a word problem:

Step-I Read the problem carefully and note what is given and what is required.

Step-II Denote the unknown quantity by some letters, say x, y, z, etc.

Step-III Translate the statements of the problem into mathematical statements.

Step-IV Using the condition(s) given in the problem, form the equation.

Step-V Solve the equation for the unknown.

Step-VI Check whether the solution satisfies the equation.

❖ EXAMPLES ❖

Ex.24 A number is such that it is as much greater than 84 as it is less than 108. Find it.

Sol. Let the number be x. Then, the number is greater than 84 by x – 84 and it is less than 108 by 108 – x.

[Given]

$\therefore$ x – 84 = 108 – x

$\Rightarrow$ x + x = 108 + 84

$\Rightarrow 2x = 192 \Rightarrow \frac{2x}{2} = \frac{192}{2} \Rightarrow x = 92$

Hence, the number is 96.

Ex.25 A number is 56 greater than the average of its third, quarter and one-twelfth. Find it.

Sol. Let the number be x. Then,

One third of x is = $\frac{1}{3}$ x, Quarter of x is = $\frac{x}{4}$,

One-twelfth of x is = $\frac{x}{12}$

Average of third, quarter and one-twelfth of

$$x \text{ is} = \frac{\left(\frac{x}{3}+\frac{x}{4}+\frac{x}{12}\right)}{3} = \frac{1}{3}\left(\frac{x}{2}+\frac{x}{4}+\frac{x}{12}\right)$$

It is given that the number x is 56 greater than the average of the third, quarter and one-twelfth of x.

$$\therefore \quad x = \frac{1}{3}\left(\frac{x}{3}+\frac{x}{4}+\frac{x}{12}\right) + 56$$

$$\Rightarrow \quad x = \frac{x}{9} + \frac{x}{12} + \frac{x}{36} + 56$$

$$\Rightarrow \quad x - \frac{x}{9} - \frac{x}{12} - \frac{x}{36} + 56$$

$\Rightarrow$ 36x – 4x – 3x – x = 36 × 56

[Multiplying both sides by 36 i.e., the L.C.M. of 9, 12 and 36]

$\Rightarrow$ 36x – 8x = 36 × 56

$\Rightarrow$ 28x = 36 × 56

$$\Rightarrow \frac{28x}{28} = \frac{36 \times 56}{28}$$

[Dividing both sides by 28]

$\Rightarrow$ x = 36 × 2

$\Rightarrow$ x = 72

Hence, the number is 72.

Ex.26 A number consists of two digits whose sum is 8. If 18 is added to the number, the digits are interchanged. Find the number

Sol. Let one's digit be x.

Since the sum of the digits is 8. Therefore, ten's digit = 8 – x.

$\therefore$ Number = 10 ×(8 – x) + x = 80 – 10x + x

= 80 – 9x ... (i)

Now,

Number obtained by reversing the digit

= 10 × x + (8 – x) = 10x + x – x = 9x + 8.

It is given that if 18 is added to the number its digits are reversed.

$\therefore$ Number + 18 = Number obtained by reversing the digits

$\Rightarrow$ 80 – 9x + 18 = 9x + 8

$\Rightarrow$ 98 – 9x = 9x + 8 $\Rightarrow$ 98 – 8 = 9x + 9x

$$\Rightarrow \quad 90 = 18x \Rightarrow \frac{18x}{18} = \frac{90}{18}$$

$\Rightarrow$ x = 5

Putting the value of x in (i), we get

Number = 80 – 9 × 5 = 80 – 45 = 35

Ex.27 Divide 34 into two parts in such a way that $\left(\frac{4}{7}\right)^{th}$ of one part is equal to $\left(\frac{2}{5}\right)^{th}$ of the other.

Sol. Let one part be x. Then, other part is (34 – x).

It is given that

$\left(\frac{4}{7}\right)^{th}$ of one part = $\left(\frac{2}{5}\right)^{th}$ of the other part

$\Rightarrow \frac{4}{7}x = \frac{2}{5}(34 - x)$ $\Rightarrow 20x = 14(34 - x)$

[Multiplying both sides by 35, the LCM of 7 and 5]

$\Rightarrow 20x = 14 \times 34 - 14x$

$\Rightarrow 20x + 14x = 14 - 34$

$\Rightarrow 34x = 14 \times 34$

$\Rightarrow \frac{34x}{34} = \frac{14 \times 34}{34}$ [Dividing both sides by 34]

$\Rightarrow x = 14$

Hence, the two parts are 14 and 34 – 14 = 20

Ex.28 The numerator of a fraction is 4 less that the denominator. If 1 is added to both its numerator and denominator, it becomes 1/2. Find the fraction.

Sol. Let the denominator of the fraction be x. Then,

Numerator of the fraction = x – 4

$\therefore$ Fraction = $\frac{x-4}{x}$ ...(i)

If 1 is added to both its numerator and denominator, the fraction becomes $\frac{1}{2}$

$\therefore \frac{x-4+1}{x+1} = \frac{1}{2}$

$\Rightarrow \frac{x-3}{x+1} = \frac{1}{2}$

$\Rightarrow 2(x - 3) = x + 1$

[Using cross-multiplication]

$\Rightarrow 2x - 6 = x + 1$

$\Rightarrow 2x - x = 6 + 1$

$\Rightarrow x = 7$

Putting x = 7 in (i), we get

Fraction = $\frac{7-4}{7} = \frac{3}{7}$

Hence, the given fraction is $\frac{3}{7}$.

Ex.29 Saurabh has Rs 34 in form of 50 paise and twenty-five paise coins. If the number of 25-paise coins be twice the number of 50-paise coins, how many coins of each kind does he have ?

Sol. Let the number of 50-paise coins be x. Then,

Number of 25-paise coins = 2x

$\therefore$ Value of x fifty-paise coins = 50 × x paise

$= \text{Rs } \frac{50 \times x}{100} = \text{Rs } \frac{x}{2}$

Value of 2x twenty-five paise coins

= 25 × 2x paise

$= \text{Rs } \frac{50 \times x}{100} = \text{Rs } \frac{x}{2}$

$\therefore$ Total value of all coins $= \text{Rs}\left(\frac{x}{2} + \frac{x}{2}\right) = \text{Rs } x$

But, the total value of the money is Rs 34

$\therefore$ x = 34

Thus, number of 50-paise coins = 34

Number of twenty-five paise coins

= 2x = 2 × 34 = 68

Ex.30 Arvind has Piggy bank. It is full of one-rupee and fifty-paise coins. It contains 3 times as many fifty paise coins as one rupee coins. The total amount of the money in the bank is ₹ 35. How many coins of each kind are there in the bank ?

Sol. Let there be x one rupee coins in the bank. Then,

Number of 50-paise coins = 3x

$\therefore$ Value of x one rupee coins = ₹ x

Value of 3x fifty-paise coins = 50 × 3x paise

= 150 x = paise = ₹ $\frac{150}{100}x$ = ₹ $\frac{3x}{2}$

$\therefore$ Total value of all the coins = ₹ $\left(x + \frac{3x}{2}\right)$

But, the total amount of the money in the bank is given as ₹ 35.

$\therefore x + \frac{3x}{2} = 35$

$\Rightarrow 2x + 3x = 70$ [Multiplying both sides by 2]

$\Rightarrow 5x = 70 \Rightarrow \frac{5x}{5} = \frac{70}{5} \Rightarrow x = 14$

$\therefore$ Number of one rupee coins = 14, Number of 50 paise coins = 3x = 3 × 14 = 42.

Ex.31 Kanwar is three years older than Anima. Six years ago, Kanwar's age was four times Anima's age. Find the ages of Knawar and Anima.

Sol. Let Anima's age be x years. Then, Kanwar's age is (x + 3) years.

Six years ago, Anima's age was (x – 6) years

It is given that six years ago Kanwar's age was four times Anima's age.

$\therefore\ x - 3 = 4(x - 6)$

$\Rightarrow\ x - 3 = 4x - 24\ \Rightarrow x - 4x = -24 + 3$

$\Rightarrow\ -3x = -21 \quad \Rightarrow \frac{-3x}{-3} = \frac{-21}{-3}$

$\Rightarrow\ x = 7$

Hence, Anima's age = 7 years

Kanwar's age = (x + 3) yers

= (7 + 3) years = 10 years.

Ex.32 Hamid has three boxes of different fruits. Box A weighs $2\frac{1}{2}$ kg more than Box B and Box C weighs $10\frac{1}{4}$ kg more than Box B. The total weight of the boxes is $48\frac{3}{4}$. How many kg does Box A weigh ?

Sol. Suppose the box B weights x kg.

Since box A weighs $2\frac{1}{2}$ kg more than box B and C weighs $10\frac{1}{4}$ kg more than box B.

$\therefore$ Weight of box A = $\left(x + 2\frac{1}{2}\right)$kg

$= \left(x + \frac{5}{2}\right)$kg ... (i)

Weight of box C = $\left(x + 10\frac{1}{4}\right)$kg

$= \left(x + \frac{41}{4}\right)$kg

$\therefore$ Total weight of all the boxes

$= \left(x + \frac{5}{2} + x + x + \frac{41}{4}\right)$kg

But, the total weight of the boxes is given as $48\frac{3}{4}$ kg = $\frac{195}{4}$ kg

$\therefore\ x + \frac{5}{2} + x + x + \frac{41}{4} = \frac{195}{4}$

$\Rightarrow\ 4x + 10 + 4x + 4x + 41 = 195$

[Multiplying both sides by 4]

$\Rightarrow\ 12x + 51 = 195$

$\Rightarrow\ 12x + 195 - 51$

$\Rightarrow\ 12x = 144$

$\Rightarrow\ \frac{12x}{12} = \frac{144}{12}$

$\Rightarrow\ x = 12$

Putting x = 12 in (i), we get

Weight of box A = $\left(12 + \frac{5}{2}\right)$kg = $14\frac{1}{2}$ kg

Ex.33 The sum of two numbers is 45 and their ratio is 7 : 8. Find the numbers.

Sol. Let one of the numbers be x. Since the sum of the two numbers is 45. Therefore, the other number will be 45 – x.

It is given that the ratio of the numbers is 7 : 8.

$\therefore\ \frac{x}{45 - x} = \frac{7}{8}$

$\Rightarrow\ 8 \times x = 7 \times (45 - x)$

[By cross-multiplication]

$\Rightarrow\ 8x = 315 - 7x\ \Rightarrow 8x + 7x = 315$

$\Rightarrow\ 15x = 315\ \Rightarrow x = \frac{315}{15} = 21$

Thus, one number is 21 and,

Other number = 45 – x = 45 – 21 = 24

Check Clearly, sum of the numbers = 21 + 24 = 45, which is same as given in the problem.

Ratio of the numbers $= \frac{21}{24} = \frac{7}{8}$ which is same as given in the problem.

Thus, our solution is correct.

Ex.34 Divide ₹ 1380 among Ahmed, John and Babita so that the amount Ahmed receives is 5 times as much as Babita's share and is 3 times as much as John's share.

Sol. Let Babita's share be ₹ x. Then,

Ahmed's share = ₹ 5x

$\therefore$ John's share = Total amount – (Babita's share + Ahmed's share)

= ₹ [1380 – (x + 5x)] = ₹ (1380 – 6x)

It is given that Ahmed's share is three times John's share.

$\therefore$ $5x = 3(1380 - 6x) \Rightarrow 5x = 4140 - 18x$

$\Rightarrow 5x + 18x = 4140 \Rightarrow 23x = 4140$

$\Rightarrow x = \frac{4140}{23} = 180$

$\therefore$ Babita's share = ₹ 180, Ahmed's share = ₹ (5 × 180) = ₹ 900

John's share = ₹ (1380 – 6 × 180) = ₹ 300

Ex.35 The length of a rectangle exceeds its breadth by 4 cm. If length and breadth are each increased by 3 cm, the area of the new rectangle will be 81 cm^2 more than that of the given rectangle. Find the length and breadth of the given rectangle.

Sol. Let the breadth of the given rectangle be x cm. Then, Length = (x + 4) cm

$\therefore$ Area = Length × Breadth = $(x + 4)x = x^2 + 4x$.

When length and breadth are each increased by 3 cm.

New length = (x + 4 + 3) cm = (x + 7) cm,

New breadth = (x + 3) cm

$\therefore$ Area of new rectangle = Length × Breadth

$= (x + 7)(x + 3)$

$= x(x + 3) + 7(x + 3)$

$= x^2 + 3x + 7x + 21 = x^2 + 10x + 21$

It is given that the area of new rectangle is 81 cm^2 more than the given rectangle.

$\therefore$ $x^2 + 10x + 21 = x^2 + 4x + 81$

$\Rightarrow x^2 + 10x - x^2 - 4x = 81 - 21$

$\Rightarrow 6x = 60 \Rightarrow x = \frac{60}{6} = 10$

Thus,

Length of the given rectangle = (x + 4)cm = (10 + 4) cm = 14 cm

Breadth of the given rectangle = 10 cm

Check Area of the given rectangle = $(x^2 + 4x)cm^2$

$= (10^2 + 4 \times 10)cm^2 = 140\ cm^2$

Area of the new rectangle

$= (x^2 + 10x + 21)cm^2$

$= (10^2 + 10 \times 10 + 21)cm^2 = 221\ cm^2$

Clearly, area of the new rectangle is 81 cm^2 more than that of the given rectangle, which is the same as given in the problem. Hence, our answer is correct.

Ex.36 An altitude of a triangle is five-thirds the length of its corresponding base. If the altitude were increased by 4 cm and the base be decreased by 2 cm, the area of the triangle would remain the same. Find the base and the altitude of the triangle.

Sol. Let the length of the base of the triangle be x cm. Then,

Altitude = $\left(\frac{5}{3} \times x\right)$ cm = $\frac{5x}{3}$ cm

$\therefore$ Area = $\frac{1}{2}$ (Base × Altitude) cm^2

$= \frac{1}{2}\left(x \times \frac{5x}{3}\right)cm^2 = \frac{5x^2}{6}\ cm^2$

When the altitude is increased by 4 cm and the base is decreased by 2 cm, we have

New base = (x – 2) cm,

New altitude = $\left(\frac{5x}{3} + 4\right)$ cm

$\therefore$ Area of the new triangle

$= \frac{1}{2}$ (Base × Altitude)

$$= \frac{1}{2}\left\{\left(\frac{5x}{3}+4\right)\times(x-2)\right\}\text{cm}^2$$

$$= \frac{1}{2}\left\{(x-2)\times\left(\frac{5x}{3}+4\right)\right\}\text{cm}^2$$

$$= \frac{1}{2}\left\{\frac{5x}{3}(x-2)+4(x-2)\right\}\text{cm}^2$$

$$= \frac{1}{2}\left\{\frac{5x^2}{3}-\frac{10x}{3}+4x-8\right\}\text{cm}^2$$

$$= \frac{1}{2}\left(\frac{5x^2}{6}-\frac{5x}{3}+2x-4\right)\text{cm}^2$$

It is given that the area of the given triangle is same as the area of the new triangle.

$$\therefore \quad \frac{5x^2}{6} = \frac{5x^2}{6} - \frac{5x}{3} + 2x - 4$$

$$\Rightarrow \quad \frac{5x^2}{6} - \frac{5x^2}{6} + \frac{5x}{3} - 2x = -4$$

$$\Rightarrow \quad \frac{5x}{3} - 2x = -4$$

[Multiplying both sides by 3]

$\Rightarrow \quad -x = -12$

$\Rightarrow \quad x = 12\text{cm}$

Hence, base of the triangle = 12 cm.

Altitude of the triangle = $\left(\frac{5}{3}\times 12\right)$ cm = 20 cm

Check We have,

Area of the given triangle

$$= \left(\frac{5}{3}\times 12^2\right)\text{cm}^2 = 120\text{cm}^2$$

Area of the given triangle

$$= \left(\frac{5}{6}\times 12^2 - \frac{15}{3}\times 12 + 2\times 12 - 4\right)\text{cm}^2$$

$= 120\text{cm}^2$

Therefore, area of the given triangle is the same as that of the new triangle, which is the same as given in the problem. Thus, our answer is correct.

❖ EXAMPLES ❖

Ex.37 The perimeter of a rectangle is 13 cm and its width is $2\frac{3}{4}$ cm. Find its length.

Sol. Assume the length of the rectangle to be x cm. The perimeter of the rectangle

$= 2 \times (\text{length} + \text{width}) = 2 \times (x + 2\frac{3}{4})$

$= 2\left(x + \frac{11}{4}\right)$

The perimeter is given to be 13 cm.

Therefore, $2\left(x+\frac{11}{4}\right) = 13$

or $\quad x + \frac{11}{4} = \frac{13}{2}$ (dividing both sides by 2)

or $\quad x = \frac{13}{2} - \frac{11}{4} = \frac{26}{4} - \frac{11}{4} = \frac{15}{4} = 3\frac{3}{4}$

The length of the rectangle is $3\frac{3}{4}$ cm

Ex.38 The present age of Sahil's mother is three times the present age of Sahil. After 5 years their ages will add to 66 years. Find their present ages.

Sol. Let Sahil's present age be x-years.

We could also choose sahil's age 5 years later to be x and proceed. Why don't you try it that way ?

	Sahil	Mother	Sum
Present age	x	3x	
Age 5 years later	x + 5	3x + 5	4x + 10

It is given that this sum is 66 years

Therefore, $\quad 4x + 10 = 66$

This equation determines sahil's present age which is x years. To solve the equation,

we transpose 10 to RHS,

or $\quad 4x = 66 - 10 \qquad 4x = 56$

or $\quad x = \frac{56}{4} = 14$

Thus, Sahil's present age is 14 years and his mother's age is 42 years. (You may easily check that 5 years from now the sum of their ages will be 66 years)

Ex.39 Bansi has 3 times as many two-rupee coins as he has five-rupee coins. If he has in all a sum of ₹77, how many coins of each denomination does he have ?

Sol. Let the number of five-rupee coins that Bansi has be x. Then the number of two-rupee coins he has is 3 times x or 3x.

The amount Bansi has :

(i) from 5 rupee coins, ₹5 × x = ₹5x

(ii) from 2 rupee coins, ₹2 × 3x = ₹6x

Hence the total money he has = ₹11x

But this is given to be ₹77; therefore,

$11x = 77$

or $x = \frac{77}{11} = 7$

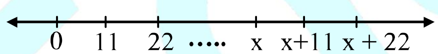
Thus, number of five-rupee coins = x = 7

and number of two-rupee coins = 3x = 21

(You can check that the total money with Bansi is ₹77)

Ex.40 The sum of three consecutive multiples of 11 is 363. Find these multiple.

Sol. If x is a multiple of 11, the next multiple is x + 11. The next to this is x + 11 + 11 or x + 22. So we can take three consecutive multiple of 11 as x,x + 11 and x + 22.



It is given that the sum of these consecutive multiples of 11 is 363. This will given the following equation :

$x + (x + 11) + (x + 22) = 363$

or $x + x + 11 + x + 22 = 363$

or $3x + 33 = 363$

or $3x = 363 - 33$

$3x = 330$

or $x = \frac{330}{3} = 110$

Ex.41 The difference between two whole numbers is 66. The ratio of the two numbers is 2 : 5. What are the two numbers ?

Sol. Since the ratio of the two numbers is 2 : 5, we may take one number to be 2x and the other to be 5x. (Note that 2x : 5x is same as 2 : 5)

The difference between the two numbers is (5x – 2x). It is given that the difference is 66. Therefore,

$5x - 2x = 66$ or $3x = 66$ or $x = 22$

Since the numbers are 2x and 5x, they are 2 × 22 or 44 and 5 × 22 or 110, respectively.

The difference between the two numbers is 110 – 44 = 66 as desired.

Ex.42 Deveshi has a total of ₹590 as currency notes in the denominations of ₹50, ₹20 and ₹10. The ratio of the number of ₹50 notes and ₹20 notes is 3 : 5. If she has a total of 25 notes, how many notes of each denomination she has ?

Sol. Let the number of ₹50 notes and ₹20 notes be 3x and 5x, respectively. But she has 25 notes in total.

Therefore, the number of ₹10 notes

$= 25 - (3x + 5x) = 25 - 8x$

The amount she has

from ₹50 notes : 3x × 50 = ₹150 x

from ₹20 notes : 5x × 20 = ₹100 x

from ₹10 notes : (25 – 8x) × 10 = ₹(250 – 80x)

Hence the total money she has

= 150x + 100 x + (250 – 80x) = ₹(170x + 250)

But she has ₹590. Therefore,

$170x + 250 = 590$

or $170x = 590 - 250 = 340$

or $x = \frac{340}{170} = 2$

The number of ₹50 notes she has = 3x

= 3 × 2 = 6

The number of ₹50 notes she has = 3x

= 3 × 2 = 6

The number of ₹20 notes she has = 5x

= 5 × 2 = 10

The number of ₹10 notes she has = 25 – 8x

= 25 – (8 × 2) = 25 – 16 = 9

Ex.43 The digits of a two-digit number differ by 3. If the digits are interchanged, and the resulting number is added to the original number, we get 143. What can be the original number ?

Sol. Let us take the two digit number such that the digit in the unit place is b. The digit in the tens place differs from b by 3. Let us take it as b + 3. So the two-digit number is

$10(b+3) + b = 10b + 30 + b = 11b + 30.$

With interchange of digits, the resulting two-digit number will be

$10b + (b+3) = 11b + 3$

If we add these two two-digit numbers, their sum is

$(11b + 30) + (11b + 3)$

$= 11b + 11b + 30 + 3 = 22b + 33$

It is given that the sum is 143. Therefore,

$22b + 33 = 143$

or $22b = 143 - 33$

or $22b = 110$

or $b = \frac{110}{22}$

or $b = 5$

The units digit is 5 and therefore the tens digit is 5 + 3 which is 8. The number is 85.

The statement of the example is valid for both 58 and 85 and both are correct answers.

Check : On interchange of digit the number we get is 58. The sum of 85 and 58 is 143 as given.

Ex.44 Arjun is twice as old as shriya. Five years ago his age was three times shriya's age. Find their present ages.

Sol. Let us take shriya's present age to be x-years.
Then Arjun's present age would be 2x years.
Shriya's age five years ago was (x – 5) years.
Arjun's age five years ago was (2x – 5) years.
It is given that Arjun's age five years ago was three times shriya's age.

Thus, $2x - 5 = 3(x - 5)$

or $2x - 5 = 3x - 15$

or $15 - 5 = 3x - 2x$

or $10 = x$

So, Shriya's present age = x = 10 years.

Therefore, Arjun's present age = $2x = 2 \times 10$ = 20 years.

Ex.45 Present ages of Anu and Raj are in the ratio 4 : 5. Eight years from now the ratio of their ages will be 5 : 6 Find their present ages.

Sol. Let the present ages of Anu and Raj be 4x years and 5x years respectively.
After eight years, Anu's age = (4x + 8) years;
After eight years, Raj's age = (5x + 8) years.
Therefore, the ratio of their ages after eight years = $\frac{4x+8}{5x+8}$

This is given to be 5 : 6

Therefore, $\frac{4x+8}{5x+8} = \frac{5}{6}$

Cross-multiplication gives

$6(4x + 8) = 5(5x + 8)$

$24x + 48 = 25x + 40$

or $24x + 48 - 40 = 25x$

or $24x + 8 = 25x$

or $8 = 25x - 24x$

or $8 = x$

Therefore, Anu's present age = 4x = 4×8 = 32 years

Raj's present age = 5x = 5×8 = 40 years