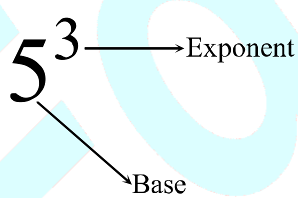
EXPONENTS & POWERS

CONTENTS

INTRODUCTION

We have learnt earlier that 5 × 5 × 5 can be written as 5^3. We read 5^3 as 'five raised to the power of three'. In 5^3, the number 5 is called the base and 3 is called the exponent or the index or the power. Similarly, exponential notation or exponential form can also be used for writing the product of a rational number multiplied by itself several times.



For example, $\frac{2}{5}\times\frac{2}{5}$ is written as $\left(\frac{2}{5}\right)^2$; $\frac{7}{4}\times\frac{7}{4}\times\frac{7}{4}$ as $\left(\frac{7}{4}\right)^3$; $\frac{-3}{2}\times\frac{-3}{2}\times\frac{-3}{2}\times\frac{-3}{2}$ as $\left(\frac{-3}{2}\right)^4$, etc.

Hence, if a is any rational number, then a × a × a × …. *m* times = a^m. Here, *a* is called the base and *m* is called the exponent.

Ex.1 Evaluate :

(a) $\left(\frac{3}{4}\right)^2$ (b) $\left(\frac{-4}{3}\right)^3$ (c) $\left(\frac{-1}{2}\right)^4$

Sol. (a) We have $\left(\frac{3}{4}\right)^2 = \frac{3}{4}\times\frac{3}{4} = \frac{9}{16}$

(b) We have $\left(\frac{-4}{3}\right)^3 = \frac{-4}{3}\times\frac{-4}{3}\times\frac{-4}{3} = \frac{-64}{27}$

(c) We have $\left(\frac{-1}{2}\right)^4 = \frac{-1}{2}\times\frac{-1}{2}\times\frac{-1}{2}\times\frac{-1}{2} = \frac{1}{16}$

Ex.2 Express the following in exponential form.

(a) $\frac{2}{5}\times\frac{2}{5}\times\frac{2}{5}$

(b) $\frac{-7}{3}\times\frac{-7}{3}\times\frac{-7}{3}\times\frac{-7}{3}$

Sol. (a) $\frac{2}{5}\times\frac{2}{5}\times\frac{2}{5} = \left(\frac{2}{5}\right)^3$

(b) $\frac{-7}{3}\times\frac{-7}{3}\times\frac{-7}{3}\times\frac{-7}{3} = \left(\frac{-7}{3}\right)^4$

Ex.3 Express in power notation.

(a) $\frac{49}{81}$ (b) $\frac{-8}{27}$

Sol. (a) $\frac{49}{81} = \frac{7\times7}{9\times9} = \frac{7}{9}\times\frac{7}{9} = \left(\frac{7}{9}\right)^2$

(b) $\frac{-8}{27} = \frac{(-2)\times(-2)\times(-2)}{3\times3\times3}$

$= \left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)$

$= \left(\frac{-2}{3}\right)^3$

Ex.4 Simplify :

(a) $\left(\frac{3}{4}\right)^2 \times \left(\frac{-2}{3}\right)^3 \times \left(\frac{-10}{12}\right)^2$

(b) $\left(\frac{-3}{5}\right)^2 \times \left(\frac{4}{9}\right)^4 \times \left(\frac{-15}{18}\right)^2$

Sol. (a) $\left(\frac{3}{4}\right)^2 \times \left(\frac{-2}{3}\right)^3 \times \left(\frac{-10}{12}\right)^2$

$= \left(\frac{3}{4}\times\frac{3}{4}\right) \times \left(\frac{-2}{3}\times\frac{-2}{3}\times\frac{-2}{3}\right) \times \left(\frac{-10}{12}\times\frac{-10}{12}\right)$

$= \frac{9}{16} \times \frac{-8}{27} \times \frac{100}{144} = \frac{-25}{216}$

(b) $\left(\frac{-3}{5}\right)^2 \times \left(\frac{4}{9}\right)^4 \times \left(\frac{-15}{18}\right)^2$

$= \left(\frac{-3}{5}\times\frac{-3}{5}\right) \times \left(\frac{4}{9}\times\frac{4}{9}\times\frac{4}{9}\times\frac{4}{9}\right) \times \left(\frac{-15}{18}\times\frac{-15}{18}\right)$

$= \frac{9}{25} \times \frac{256}{6561} \times \frac{225}{324} = \frac{64}{6561}$

Ex.5 Find the reciprocal of the following :

(a) 3^2 (b) $(-4)^3$ (c) $\left(\frac{3}{4}\right)^3$

Sol. (a) Reciprocal of $3^2 = \frac{1}{3^2} = \frac{1}{9}$

(b) Reciprocal of $(-4)^3$

$= \left(\frac{1}{-4}\right)^3 = \frac{1}{-4}\times\frac{1}{-4}\times\frac{1}{-4} = \frac{1}{-64}$

(c) Reciprocal of $\left(\frac{3}{4}\right)^3$

$= \left(\frac{4}{3}\right)^3 = \frac{4}{3}\times\frac{4}{3}\times\frac{4}{3} = \frac{64}{27}$

➤ LAWS OF EXPONENTS (Positive Exponents)

There are certain laws that govern the operations in numbers which are expressed in the exponential notation.

Law-I If x is a rational number and m and n are positive integers, then $x^m \times x^n = x^{m+n}$.

For Example, $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^3$

$= \left(\frac{2}{3}\times\frac{2}{3}\right)\times\left(\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\right) = \left(\frac{2}{3}\right)^{2+3} = \left(\frac{2}{3}\right)^5$

Law-II If x is a rational number and m and n are positive integers, such that m > n, then $x^m \div x^n = x^{m-n}$

For Example, $\left(\frac{4}{5}\right)^5 \div \left(\frac{4}{5}\right)^3 = \left(\frac{4}{5}\right)^5 \times \left(\frac{5}{4}\right)^3$

$= \frac{4}{5}\times\frac{4}{5}\times\frac{4}{5}\times\frac{4}{5}\times\frac{4}{5}\times\frac{5}{4}\times\frac{5}{4}\times\frac{5}{4}$

$= \frac{4}{5}\times\frac{4}{5} = \left(\frac{4}{5}\right)^2$ or $\left(\frac{4}{5}\right)^{5-3}$

Law-III If x is a rational number and m and n are positive integers, then $(x^m)^n = x^{m\times n} = x^{mn}$.

For Example, $(2^2)^3 = 2^2 \times 2^2 \times 2^2 = 2^{2+2+2}$

$= 2^6 = 2^{2\times 3}$ (From Law I)

Law-IV If $\frac{x}{y}$ is any rational number and *m* is any positive integer, then $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$.

For Example, $\left(\frac{2}{7}\right)^3 = \frac{2}{7}\times\frac{2}{7}\times\frac{2}{7} = \frac{2^3}{7^3}$

Law-V

1. **The Zero Exponent**: If x is a rational number and $x \neq 0$, then $x^0 = 1$.

   For example, $5^0 = 1$ and $(-7)^0 = 1$.

2. **The Negative Exponent**: If x is a rational number different from 0, then x^{-1} denotes the reciprocal of x. We know that the reciprocal of x is $\frac{1}{x}$. Therefore, $x^{-1} = \frac{1}{x}$.

   For Example, $6^{-1} = \frac{1}{6}$ and $11^{-1} = \frac{1}{11}$.

Law VI If x is any rational number different from 0 and m is a positive integer, then x^{-m} denotes the reciprocal of x^m.

Also, $\left(\frac{x}{y}\right)^{-m} = \left(\frac{y}{x}\right)^{m}$.

For Example, $\left(\frac{2}{3}\right)^{-3} = \frac{1}{\left(\frac{2}{3}\right)^3}$

$$= \frac{1}{\frac{2^3}{3^3}} = \frac{3^3}{2^3} = \left(\frac{3}{2}\right)^3$$

Ex.6 Find the value of $\left(\frac{-3}{4}\right)^3 \times \left(\frac{-3}{4}\right)^4$.

Sol. The numbers $\left(\frac{-3}{4}\right)^3$ and $\left(\frac{-3}{4}\right)^4$ have the same bases. So to find the product, we add their powers.

Thus, $\left(\frac{-3}{4}\right)^3 \times \left(\frac{-3}{4}\right)^4 = \left(\frac{-3}{4}\right)^{3+4} = \left(\frac{-3}{4}\right)^7$

$$= \frac{-2187}{16384}$$

Ex.7 Evaluate : $\left(\frac{3}{4}\right)^6 \div \left(\frac{3}{4}\right)^4$

Sol. The numbers $\left(\frac{3}{4}\right)^6$ and $\left(\frac{3}{4}\right)^4$ have the same bases. So, to find the solution, we have to subtract the powers.

$$\therefore \left(\frac{3}{4}\right)^6 \div \left(\frac{3}{4}\right)^4 = \left(\frac{3}{4}\right)^{6-4} = \left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$$

Ex.8 Simplify :

(a) $\left(\frac{2}{3}\right)^{-3} \times \left(\frac{2}{3}\right)^{-1}$ (b) $\left[\left(\frac{-1}{2}\right)^{-3}\right]^{-2}$

Sol. (a) The given product can be written as -

$$\left(\frac{2}{3}\right)^{-3} \times \left(\frac{2}{3}\right)^{-1} = \left(\frac{2}{3}\right)^{-3+(-1)}$$

$$= \left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^{4} = \frac{81}{16}$$

(b) The exponent $\left[\left(\frac{-1}{2}\right)^{-3}\right]^{-2}$ can be written as

$$\left(\frac{-1}{2}\right)^{[(-3)\times(-2)]} = \left(\frac{-1}{2}\right)^{6} = \frac{1}{64}$$

Ex.9 If $\frac{p}{q} = \left(\frac{3}{2}\right)^{-2} \div \left(\frac{6}{7}\right)^{0}$, find the value of $\left(\frac{p}{q}\right)^{-3}$.

Sol. We have $\left(\frac{p}{q}\right) = \left(\frac{3}{2}\right)^{-2} \div \left(\frac{6}{7}\right)^{0} = \left(\frac{2}{3}\right)^{2} \div 1$

$$= \left(\frac{2}{3}\right)^{2} \times 1 = \left(\frac{2}{3}\right)^{2}$$

$$\therefore \left(\frac{p}{q}\right)^{-3} = \left[\left(\frac{2}{3}\right)^{2}\right]^{-3} = \left(\frac{2}{3}\right)^{-6} = \left(\frac{3}{2}\right)^{6}$$

Ex.10 If $5^{2x+1} \div 25 = 125$, find the value of x.

Sol. We have $5^{2x+1} \div 25 = 125$

or $\frac{5^{2x+1}}{25} = 125$ or $\frac{5^{2x+1}}{5\times5} = 5 \times 5 \times 5$

or $\frac{5^{2x+1}}{5^2} = 5^3$ or $5^{2x+1-2} = 5^3$

or $5^{2x-1} = 5^3$ or $2x - 1 = 3$

or $2x = 4$ or $x = 2$

Ex.11 Simplify and express the result as a power of 2.

(a) $(3^4 \times 3^5) \div 3^9$ (b) $\left[\left(\frac{1}{5}\right)^6 \div \left(\frac{1}{5}\right)^5\right] \div \frac{1}{5}$

Sol. (a) We have $(3^4 \times 3^5) \div 3^9 = 3^{4+5} \div 3^9 = 3^9 \div 3^9$
$= 3^{9-9} = 3^0 = 1 = 2^0$,
Where 1 can be expressed as a power of 2 as 2^0.

(b) $\left[\left(\frac{1}{5}\right)^6 \div \left(\frac{1}{5}\right)^5\right] \div \frac{1}{5} = \left[\frac{\left(\frac{1}{5}\right)^6}{\left(\frac{1}{5}\right)^5}\right] \div \frac{1}{5}$

$= \left[\left(\frac{1}{5}\right)^{6-5}\right] \div \frac{1}{5}$

$= \left(\frac{1}{5}\right)^1 \div \left(\frac{1}{5}\right)^1 = \left(\frac{1}{5}\right)^{1-1} = \left(\frac{1}{5}\right)^0 = 1 = 2^0$

Again, 1 can be expressed as the power of 2 as 2^0.

Ex.12 By what number should we multiply 7^{-5}, so that the product may be equal to 7 ?

Sol. Let the number be x.

Then $7^{-5} \times x = 7$

or $x = \frac{7}{7^{-5}} = 7 \times 7^5 = 7^6$

$\therefore$ The number is 7^6.

Ex.13 By what number should 7^5 be divided, so that the quotient is 7^{-3} ?

Sol. Let the number be x.

Then $7^5 \div x = 7^{-3}$

or $\frac{7^5}{x} = 7^{-3}$ or $\frac{7^5}{7^{-3}} = x$

or $x = 7^5 \times 7^3 = 7^8$

$\therefore$ The number is 7^8.

➢ STANDARD AND USUAL FORMS OF NUMBER

Observe the following facts.

1. The distance from the Earth to the sun is 149,600,000,000 m.
2. The speed of light is 300,000,000 m/sec.
3. Thickness of Class VII Mathematics book is 20 mm.
4. The average diameter of a Red Blood Cell is 0.000007 m.
5. The thickness of human hair is in the range of 0.005 cm to 0.01 cm.
6. The distance of moon from the Earth is 384,467,000 m (approx).
7. The size of a plant cell is 0.00001275 m.
8. Average radius of the Sun is 695000 km.
9. Mass of propellant in a space shuttle solid rocket booster is 503600 kg.
10. Thickness of a piece of paper is 0.0016 cm.
11. Diameter of a wire on a computer chip is 0.000003 m.
12. The height of Mount Everest is 8848 m.

Observe that there are few numbers which we can read like 2 cm, 8848 m, 6,95,000 km. There are some large numbers like 150,000,000,000 m and some very small numbers like 0.000007 m. Identify very large and very small numbers from the above facts and write them in the adjacent table.

Very large numbers	Very small numbers
150,000,000,000 m	0.000007 m
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for example : $150,000,000,000 = 1.5 \times 10^{11}$ (standard form). Now, let us try to express 0.000007 m (usual form) in standard form.

$0.000007 = \frac{7}{1000000} = \frac{7}{10^6} = 7 \times 10^{-6}$

$0.000007 \text{ m} = 7 \times 10^{-6} \text{ m}$

Note : $\underset{11\,10\,9\,8\,7\,6\,5\,4\,3\,2\,1}{1\underbrace{50000000000}}$ decimal is moved 11 placed to the left.

Similarly, consider the thickness of a piece of paper which is 0.0016 cm.

$$0.0016 = \frac{16}{10000} = \frac{1.6\times10}{10^4} = 1.6 \times 10 \times 10^{-4}$$

Note : $0.\underset{1\,2\,3\,4\,5\,6}{\underbrace{000007}}$ decimal is moved 6 places to the right.

Therefore, we can say thickness of paper is 1.6×10^{-3} cm

Note : $0.\underset{1\,2\,3}{\underbrace{0016}}$ decimal is moved 3 places to the right.

➤ COMPARE OF NUMBERS

The diameter of the Sun is 1.4×10^9 m and the diameter of the Earth is 1.2756×10^7 m. Suppose you want to compare the diameter of the Earth, with the diameter of the Sun.

Diameter of the Sun = 1.4×10^9 m

Diameter of the Earth = 1.2756×10^7 m

Therefore $\frac{1.4\times10^9}{1.2756\times10^7} = \frac{1.4\times10^{9-7}}{1.2756} = \frac{1.4\times100}{1.2756}$

which is approximately 100 So, the diameter of the Sun is about 100 times the diameter of the earth.

Let us compare the size of a Red blood cell which is 0.000007 m to that of a plant cell which is 0.00001275 m.

Size of Red Blood cell = 0.000007 m = 7×10^{-6} m

Size of plant cell = 0.00001275 = 1.275×10^{-5} m

Therefore, $\frac{7\times10^{-6}}{1.275\times10^{-5}} = \frac{7\times10^{-6-(-5)}}{1.275}$

$= \frac{7\times10^{-1}}{1.275} = \frac{0.7}{1.275} = \frac{0.7}{1.3} = \frac{1}{2}$ (approx.)

So a red blood cell is half of plant cell in size.

Mass of earth is 5.97×10^{24} kg and mass of moon is 7.35×10^{22} kg. What is the total mass ?

Note :– When we have to add numbers in standard form, we convert them into numbers with the same exponents.

Total mass = 5.97×10^{24} kg + 7.35×10^{22} kg.

$= 5.97 \times 100 \times 10^{22} + 7.35 \times 10^{22}$

$= 597 \times 10^{22} + 7.35 \times 10^{22}$

$= (597 + 7.35) \times 10^{22}$

$= 604.35 \times 10^{22}$ kg

Eg. The distance between Sun and Earth is 1.496×10^{11}m and the distance between Earth and Moon is 3.84×10^8 m.

During solar eclipse moon comes in between Earth and Sun. At that time what is the distance between Moon and Sun.

Distance between Sun and Earth = 1.496×10^{11} m

Distance between Earth and Moon = 3.84×10^8 m

Distance between Sun and Moon

$= 1.496 \times 10^{11} - 3.84 \times 10^8$

$= 1.496 \times 1000 \times 10^8 - 3.84 \times 10^8$

$= (1496 - 3.84)\times10^8$ m $= 1492.16\times10^8$ m

Ex.14 Express the following numbers in standard form.

(i) 0.000035 (ii) 4050000

Sol. (i) $0.000035 = 3.5 \times 10^{-5}$

(ii) $4050000 = 4.05 \times 10^6$

Ex.15 Express the following numbers in usual form.

(i) 3.52×10^5 (ii) 7.54×10^{-4}

(iii) 3×10^{-5}

Sol. (i) $3.52 \times 10^5 = 3.52 \times 100000 = 352000$

(ii) $7.54 \times 10^{-4} = \frac{7.54}{10^4} = \frac{7.54}{10000} = 0.000754$

(iii) $3 \times 10^{-5} = \frac{3}{10^5} = \frac{3}{100000} = 0.00003$