COMPARING QUANTITIES

CONTENTS

➢ PERCENTAGE

A fraction with its denominator as 100 is called percent, the symbol '%' is used for percent and it indicates multiplication with $\frac{1}{100}$.

For example,

$$\frac{18}{100} = 18 \text{ hundredths}$$
$$= 18 \text{ per hundred}$$
$$= 18 \times \frac{1}{100}$$
$$= 18 \text{ percent}$$
$$= 18\%$$

$\frac{18}{100}$ can also be expressed as 18 : 100.

30% is equivalent to the ratio 30 : 100.

Or

30% is equivalent to the fraction $\frac{30}{100}$ or $\frac{3}{10}$.

Working Rules

To Find the Percentage of a Number

To find the value of a given percent of a given quantity, we multiply the given quantity by the fraction or decimal fraction of the given percent, i.e., **value of a given percent = Given quantity × given percent converted into fraction**.

❖ EXAMPLES ❖

Ex.1 If 8.5% of a number is 51, then find the number.

Sol. Let the required number be x.

$\therefore$ 8.5% of x = 51

or $\frac{85}{10}$ % of x = 51

or $\frac{85}{10} \times \frac{1}{100} \times x = 51$

or $x = \frac{\cancel{51}^{3} \times 100 \times \cancel{10}^{2}}{\cancel{85}_{\cancel{5}_{1}}} = 3 \times 100 \times 2 = 600$

Thus, 8.5% of 600 is 51.

Ex.2 The difference between increasing a number by 8% and decreasing it by 7% is 75. What is the number ?

Sol. Let the required number be x.

$\therefore$ 8% of x = $\frac{8x}{100}$

Therefore, Increased number = $x + \frac{8x}{100} = \frac{108x}{100}$

Similarly, the number decreased by

$$7\% = x - \frac{7x}{100} = \frac{93x}{100}$$

Now, $= \frac{108x}{100} - \frac{93x}{100} = \frac{15x}{100}$

But, actual difference = 75

So, $\frac{15x}{100} = 75$

Thus, $x = \frac{75 \times 100}{15}$

$= 5 \times 100 = 500$

Hence, the required number is 500.

Ex.3 Rani's weight is 25% that of Meena's weight and 40% that of Tara's weight. What percentage of Tara's weight is Meena's weight ?

Sol.: Let Meena's weight be x kg and Tara's weight be y kg.

Then, Rani's weight = 25% of Meena's weight

$$= \frac{25}{100} \times x \qquad \ldots (i)$$

Also, Rani's weight = 40% of Tara's weight

$$= \frac{40}{100} \times y \qquad \ldots (ii)$$

From (i) and (ii), we get

$$\frac{25}{100} \times x = \frac{40}{100} \times y$$

$\Rightarrow$ 25x = 40 y [Multiplying both sides by 100]

$\Rightarrow$ 5x = 8y [Dividing both sides by 5]

$$\Rightarrow x = \frac{8}{5} y \qquad \ldots (iii)$$

We have to find Meena's weight as the percentage of Tara's weight, i.e.,

$$\frac{x}{y} \times 100 = \frac{\frac{8}{5}y}{y} \times 100 \qquad \text{[Using (iii)]}$$

$$= \frac{8}{5} \times 100 = 160$$

Hence, Meena's weight is 160% of Tara's weight.

Ex.4 Rakesh's income is 25% more than that of Rohan's income. What percent is Rohan's income less than Rakesh's income ?

Sol. Let Rohan's income be Rs 100. Then,

Rakesh's income = Rs 125.

If Rakesh's income is

Rs 125, Rohan's income = Rs 100

If Rakesh's income is

Rs 1, Rohan's income = Rs $\frac{100}{125}$

If Rakesh's income is Rs 100,

Rohan's income = Rs $\left(\frac{100}{125} \times 100\right)$

= Rs 80.

Now, difference between Rohan's and Rakesh's income= Rs (100 – 80)

= Rs 20

Hence, Rohan's income is 20% less than that of Rakesh.

Ex.5 The price of sugar goes up by 20%. By how much percent must a house wife reduce her consumption so that the expenditure does not increase ?

Sol. Let the consumption of sugar originally be 100 kg and its price be Rs 100. Then,

New price of 100 kg sugar = Rs 120

[$\because$ Price increases by 20%]

Now, Rs 120 can fetch 100 kg sugar.

$\therefore$ Rs 100 can fetch = $\left(\frac{100}{120} \times 100\right)$ kg sugar

$$= \frac{250}{3} \text{ kg sugar}$$

$\therefore$ Reduction in consumption = $\left(100 - \frac{250}{3}\right)\%$

$$= \frac{50}{3}\% = 16\frac{2}{3}\%$$

Ex.6 A number is increased by 10% and then it is decreased by 10%. Find the net increase or decrease percent.

Sol: Let the number be 100.

Increase in the number = 10% of 100

$$= \frac{10}{100} \times 100 = 10$$

$\therefore$ Increased number = 100 + 10 = 110.

This number is then decreased by 10%.

Therefore, decrease in the number = 10% of 110

$$= \left(\frac{10}{100} \times 110\right) = 11.$$

$\therefore$ New number = 110 – 11 = 99

Thus, net decrease = 100 – 99 = 1

Hence, net percentage decrease = $\left(\frac{1}{100} \times 100\right)\%$

= 1%.

Ex.7 The salary of an officer has been increased by 50%. By what percent the new salary must be reduced to restore the original salary ?

Sol. Let original salary be Rs 100.

Then, Increase in the salary = 50% of Rs 100
= Rs 50.

Salary after increment = Rs 150.

Now, in order to restore the original salary, a reduction of Rs 50 should be made on Rs 150.

Thus, Reduction on Rs 150 = Rs 50

$\Rightarrow$ Reduction on Rs 1 = Rs $\frac{50}{150}$

$\Rightarrow$ Reduction on Rs 100 = Rs $\left(\frac{50}{150}\times 100\right)$

$= 33\frac{1}{3}$

$\therefore$ Reduction on new salary $= 33\frac{1}{3}\%$.

➢ PROFIT AND LOSS

Profit and Loss :

In our daily routine, we have to buy some articles from various shops. The shopkeepers purchase these articles either from wholesalers or directly from the manufacturers by paying a certain price. Generally, the shopkeeper sells his articles at a different price. These prices and difference in these prices are given special names such as cost price, selling price, profit, loss etc.

Cost Price :

The price for which an article is purchased is called the cost price and abbreviated as C.P.

Selling Price :

The price for which an article is sold is called the selling price and abbreviated as S.P.

Profit :

If selling price is more than cost price, then the difference between selling price and the cost price is called the profit.

$\therefore$ Profit = Selling Price – Cost Price

Loss :

If selling price is less than cost price, then the difference between the selling price and cost price is called loss.

$\therefore$ Loss = Cost Price – Selling Price

Overheads :

Usually, a merchant has to spend some money on freight or transport, labour or maintenance of the purchased articles. These extra expenditures are called overheads. The overheads are an essential part of cost price.

$\therefore$ Cost Price = (Payment made while purchasing the articles) + overhead charges

Some useful Formulae to Find the above defined Terms :

A. Profit or Gain (S.P. > C.P.)

1. Profit = S.P. – C.P.
2. S.P. = Profit + C.P.
3. C.P. = S.P. – Profit
4. Profit % = $\frac{\text{Profit}}{\text{C.P.}} \times 100$
5. Profit = $\frac{\text{C.P.}\times \text{Profit}\%}{100}$
6. S.P. = C.P. $\left(\frac{100+\text{Profit}\%}{100}\right)$
7. C.P. = $\left(\frac{100\times \text{S.P.}}{100+\text{Pr ofit}\%}\right)$

B. Loss (S.P. > C.P.)

1. Loss = C.P. – S.P.
2. S.P. = C.P. – Loss
3. C.P. = Loss + S.P.
4. Loss % = $\frac{\text{Loss}}{\text{C.P.}} \times 100$
5. Loss = $\frac{\text{C.P.}\times \text{Loss}\%}{100}$
6. S.P. = C.P. $\left(\frac{100-\text{Loss}\%}{100}\right)$
7. C.P. = $\frac{100\times \text{S.P.}}{(100-\text{Loss}\%)}$

❖ EXAMPLES ❖

Ex.8 Anshul purchased 100 oranges at the rate of Rs 2 per orange. He sold 60% of the oranges at the rate of Rs 2.50 per orange and the remaining oranges at the rate of Rs 2 per orange. Find his profit percent.

Sol. S.P. of 100 oranges = Rs 2 × 100 = Rs 200

$$60\% \text{ of } 100 \text{ oranges} = \frac{60}{100} \times 100 \text{ oranges}$$

= 60 oranges

Now S.P. of 60 oranges = Rs 2.50 × 60 = Rs 150

and S.P. of the remaining (100 – 60), i.e.,

40 oranges = Rs 2 × 40 = Rs 80

∴ S.P. of all the 100 oranges

= Rs 150 + Rs 80 = Rs 230

Therefore, profit = S.P. – C.P.

= Rs (230 – 200) = Rs 30

$$\text{Hence, Profit percent} = \frac{30}{200} \times 100 = 15\%$$

Thus, Anshul's profit is 15%.

Ex.9 By selling 144 eggs, Anuj lost the S.P. of 6 eggs. Find his loss percent.

Sol. Let S.P. of 1 egg = Rs 1

∴ S.P. of 144 eggs = Rs 144 × 1 = Rs 144

and, Loss = S.P. of 6 eggs

= Re 1 × 6 = Rs 6

∴ C.P. of 144 eggs = S.P. + Loss

= Rs 144 + Rs 6 = Rs 150

$$\text{Therefore, loss \%} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

$$= \frac{6}{150} \times 100 = 4$$

Thus, Anuj's loss is 4%.

Ex.10 Mahender bought two cows at Rs 20,000 each. He sold one cow at 15% gain. But he had to sell the second cow at a loss. If he had suffered a loss of Rs 1,800 on the whole dealing, find the selling price of the second cow.

Sol. Total C.P. of the two cows = 2 × Rs 20000

= Rs 40000

Loss = Rs 1800

∴ Total S.P. = Rs 40000 – Rs 1800

= Rs 38200 ... (i)

Now, S.P. of the first cow at 15% profit

$$= \text{C.P.} \left(\frac{100 + \text{Profit\%}}{100}\right)$$

$$= \text{Rs } 20000 \times \frac{(100+15)}{100}$$

$$= \text{Rs } 20000 \times \frac{115}{100}$$

= Rs 200 × 115

= Rs 23000 ... (ii)

∴ S.P. of the second cow = Rs 38200 – Rs 23000 [From (i) and (ii)]

= Rs 15200

Thus, the selling price of the second cow is Rs 15,200.

Ex.11 A man buys 60 pens at Rs 10 per pen and sells 40 pens at Rs 12 per pen and remaining 20 pens at Rs 9 per pen. Find his gain or loss percent.

Sol. Cost of 60 pens = Rs 10 × 60 = Rs 600

S.P. of 40 pens = Rs 12 × 40 = Rs 480

S.P. of 20 pens = Rs 9 × 20 = Rs 180

⇒ Total S.P. = Rs 480 + Rs 180 = Rs 660

Since, S.P. > C.P.

∴ Profit = Rs 660 – Rs 600 = Rs 60

$$\therefore \text{Profit percent} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$= \frac{60}{600} \times 100 = 10\%.$$

Ex.12 By selling an air-cooler for Rs 6,800, Mr. Avinash lost 15%. For what price should he sell it to get a profit of 10% ?

Sol. This sum will be solved in two parts. In 1st part, we find the C.P. and in 2nd part, we find the required S.P.

Part I :

S.P. of the air cooler = Rs 6800

Loss = 15% i.e., for every Rs 100 he is losing Rs 15.

∴ If C.P. is Rs 100,

then S.P. Rs 100 – Rs 15 = Rs 85

∴ If S.P. is Rs 85, then C.P. = Rs 100

$$\text{If S.P. is Rs 1, then C.P.} = \text{Rs } \frac{100}{85}$$

$$\text{If S.P. is Rs 6800, then C.P.} = \text{Rs } \frac{100}{\cancel{85}_{5}} \times \overset{80}{\cancel{6800}}_{\cancel{400}}$$

= Rs 100 × 80

= Rs 8000.

Part II :

C.P. = Rs 8000

Profit = 10%

∴ Profit = 10% of Rs 8000

$$= \frac{10}{100} \times 8000 = \text{Rs } 800$$

∴ S.P. = C.P. + Profit

= Rs 8000 + Rs 800 = Rs 8800.

Hence, the air-cooler should be sold for Rs 8,800 in order to make a profit of 10%.

Ex.13 A man sold two scooters for Rs 18000 each. On one, he gained 20% and on the other, he lost 20%. Find his total loss or gain.

Sol. S.P. of the first scooter = Rs 18000

Gain = 20%

Therefore, $C.P. = \frac{100 \times S.P.}{(100 + Profit\%)}$

$= Rs \frac{100 \times 18000}{(100 + 20)}$

$= Rs \frac{100 \times 18000}{120}$

= Rs 100 × 150

= Rs 15000 ... (i)

S.P. of the second scooter = Rs 18000

Loss = 20%

Therefore, $C.P. = \frac{100 \times S.P.}{(100 - Loss\%)}$

$= Rs \frac{100 \times 18000}{(100 - 20)}$

$= \frac{100 \times 18000}{80}$

= Rs 100 × 225

= Rs 22500 ... (ii)

Now, total C.P. = Rs 15000 + Rs 22500

[From (i) and (ii)]

= Rs 37500

and total S.P. = 2 × Rs 18000 = Rs 36000

Hence, loss = C.P. – S.P.

= Rs 37500 – Rs 36000

= Rs 1,500.

Ex.14 The cost price of 10 tables is equal to the selling price of 8 tables. Find the loss or profit percent.

Sol. Let the C.P. of each table = Rs 100

∴ C.P. of 10 tables = Rs 1000

∴ S.P. of 8 tables = Rs 1000

So, S.P. of 1 table = $Rs \frac{1000}{8}$ = Rs 125

∴ Profit on 1 table = Rs 125 – Rs 100 = Rs 25

or Profit percent = $\frac{25}{100} \times 100 = 25\%$.

➢ DISCOUNT

We read advertisements in our day-to-day life in newspapers. magazines, banners, posters given by various companies and shopkeepers declaring discounts such as :

"Off Season Discount",

"Grand Puja Discount",

"Goods at Throw away prices",

"Now get 1100 g Desi Ghee for the cost of just 1 kg.",

"Get a Steel Glass free with every 500 g pack of tea", etc.

When discount is given, a certain price is attached to the article which the shopkeeper professes to be the cost of the article for the customer. This price is called the marked price (or list price). Then, the shopkeeper offers discount on this marked price. Customer pays the difference between the marked price and the discount.

Some useful formulae regarding Discount, Marked Price, Selling Price, etc.

1. Net Selling Price = Marked Price – Discount
2. Discount = Marked Price – Net Selling Price
3. Marked Price = Net Selling Price + Discount
4. $Discount\% = \left(\frac{Discount}{Marked\ Price}\right) \times 100\%$
5. $S.P. = M.P. - \frac{Discount\% \times M.P.}{100}$
6. $S.P. = M.P. \left(1 - \frac{Discount\%}{100}\right)$
7. $S.P. = M.P. \left(\frac{100 - Discount\%}{100}\right)$
8. $M.P. = \frac{100 \times S.P.}{(100 - Discount\%)}$

Let us now consider some examples to illustrate the above facts.

❖ EXAMPLES ❖

Ex.15 Marked price of a pen is Rs 20. It is sold at a discount of 15%. Find the discount allowed on the pen and its selling price.

Sol. Marked Price of the pen = Rs 20
Rate of discount = 15%
∴ Discount allowed = 15% of Rs 20
$= \frac{15}{100} \times \text{Rs } 20 = \text{Rs } 3$
Therefore, selling price of the pen
= Rs 20 – Rs 3 = Rs 17.

Ex.16 A chain with marked price Rs 1,200 was sold to a customer for Rs 1,000. Find the rate of discount allowed on the chain.

Sol. Marked Price = Rs 1200
Selling Price = Rs 1000
Discount = Rs 1200 – Rs 1000 = Rs 200
$\text{Rate of discount} = \frac{\text{Discount}}{\text{M.P.}} \times 100\%$
$= \frac{200}{1200} \times 100\% = 16.66\%$

Ex.17 A shopkeeper offers 15% season discount to the customers and still makes a profit of 19%. What is the cost price for the shopkeeper on a saree marked at Rs 2,240 ?

Sol. M.P. = Rs 2240
Rate of discount = 15%
$\text{Discount allowed} = \text{Rs } \frac{15}{100} \times 2240 = \text{Rs } 336$
Thus, S.P. of the saree = Rs (2240 – 336)
= Rs 1904
Now, profit % of the shopkeeper = 19%
$\text{Therefore, C.P.} = \frac{100 \times \text{S.P.}}{(100 + \text{Profit}\%)}$
$= \text{Rs } \frac{100 \times 1904}{(100+19)}$
$= \text{Rs } \frac{100 \times 1904}{119}$
= Rs 100 × 16 = Rs 1600
Thus, the cost price of the saree is Rs 1,600.

Ex.18 A Jacket was sold for Rs 680 after allowing a discount of 15% on the marked price. Find the marked price of the Jacket.

Sol. Let M.P. be Rs x.
∴ Discount = 15% on Rs x
$= \text{Rs } \frac{15}{100} \times x = \text{Rs } \frac{3x}{20}$
$\therefore \text{S.P.} = \text{Rs } \left(x - \frac{3x}{20}\right) = \text{Rs } \left(\frac{20x - 3x}{20}\right)$
$= \text{Rs } \frac{17x}{20}$
According to the given condition,
$\frac{17x}{20} = 680$
or $x = \frac{680 \times 20}{17} = \text{Rs } 800$
Thus, marked price of the Jacket is Rs 800.

Ex.19 Abbas and Tony run a ready-made garments shop. They mark the garments at such a price that even after allowing a discount of 12.5%, gain a profit of 25%. Find the marked price of a ladies suit which costs them Rs 2,100.

Sol. First method : C.P. of a suit = Rs 2100
Profit = 25% of Rs 2100
$= \text{Rs } \frac{25}{100} \times 2100 = \text{Rs } 525$
∴ S.P. of the suit = Rs (2100 + 525)
= Rs 2625
Let the marked price be Rs 100.
Then, Discount = 12.5% of Rs 100
$= \frac{12.5}{100} \times 100 = \text{Rs } 12.50$
∴ S.P. = Rs (100 – 12.50) = Rs 87.50
Now, if S.P. is Rs 87.50, M.P. = Rs 100
$\therefore \text{If S.P. is Rs 2625, M.P.} = \text{Rs } \frac{100}{87.50} \times 2625$
$= \frac{100 \times 2625 \times 100}{8750}$
= Rs 3000
Thus, the marked price of the ladies suit is Rs 3,000.

Alternate Method : (after S.P. in above)
Let the marked price be Rs x.
$\text{We have} \quad \text{M.P.} = \frac{100 \times \text{S.P.}}{(100 - \text{Discount}\%)}$
$\text{or} \quad x = \frac{100 \times 2625}{(100 - 12.5)} = \frac{262500}{87.5}$
= Rs 3000
Thus, the marked price of the suit is Rs 3,000

➢ SALES TAX

Sales tax is an indirect tax. In purchasing of some specified items from the market, we have to pay a certain extra amount (at a rate specified by the Government), in addition to the cost of the item. This additional amount is called sales tax.

Working Rules

Sales tax is calculated on the selling price in the same way as we calculate percentage.

❖ EXAMPLES ❖

Ex.20 Amar buys a pair of shoes costing Rs 470. If the rate of sales tax is 7%, calculate the total amount payable by him for shoes.

Sol. Rate of sales tax = 7%

$$\text{Sales tax} = \text{Rs } \frac{7}{100} \times 470$$

$$= \frac{3290}{100} = \text{Rs } 32.90$$

Hence, total amount to be paid = Rs 470 + Rs 32.90

= Rs 502.90

Ex.21 Rakesh purchased a cycle for Rs 660 including sales tax. If the rate of sales tax is 10%, find the selling price of cycle.

Sol. Let the selling price be Rs x.

Sales tax = 10% of x

$$= \frac{10}{100} \times x = \frac{x}{10}$$

∴ Amount to be paid for the cycle

$$= x + \frac{x}{10} = \frac{11x}{10}$$

Now, $\frac{11x}{10} = 660$ (given)

Therefore, $x = \frac{660 \times 10}{11} = 600$

Hence, the selling price of the cycle is Rs 600.

Ex.22 Nazim purchases a motorcycle, having marked price of Rs 46,000 at a discount 5%. If sales tax is charged at 10%, find the amount Nazim has to pay to purchase the motorcycle.

Sol. Marked price of motor cycle = Rs 46000

Discount = 5%

∴ Discounted price of motorcycle

$$= \text{Rs } \left(46000 - 46000 \times \frac{5}{100}\right)$$

= Rs (46000 – 2300) = Rs 43700

Sales tax on Rs 43700 = Rs 43700 × $\frac{10}{100}$ = Rs4370

Amount, Nazim has to pay for motorcycle

= Rs (43700 + 4370)

= Rs 48,070.

➢ COMPOUND INTEREST

When we borrow money from a financial agency (bank, financial agency or individual), it is called the lender.

The borrowed money is called the principal.

We have to pay some additional money together with the borrowed money for a certain time period, for the benefit of using his or her money. The additional money that we pay is called the interest.

If the principal remains the same for the whole loan period (or time), then the interest is called the simple interest.

The interest together with the principal is called the amount.

If the principal does not remain the same for the whole loan period due to addition of (compounding of) interest to the principal after a certain interval of time to form the new principal, then the interest so obtained is called the compound interest.

Simple Interest :

(i) Simple Interest

$$= \frac{\text{Principal} \times \text{Rate of interest} \times \text{Time}}{100}$$

(ii) Amount = Principal + Simple Interest.

Compound Interest :

To understand compound interest, we consider the following example -

"A man lends Rs 5,000 to a finance company at 10% per annum. What interest does he get after one year ? What will be the amount then ? At the end of the year, if he decides to deposit the whole sum (amount after one year) for another year, what interest does he get at the end of the second year ?"

$$\text{Interest after one year} = \text{Rs } \frac{5000 \times 1 \times 10}{100}$$

= Rs 500

∴ Amount after one year = Rs 5000 + Rs 500

= Rs 5500.

When the deposit is Rs 5,500 in the company for one more year, the amount of Rs 5,500 due at the end of first year becomes the principal for the second year.

∴ Interest at the end of the second year

$$= \text{Rs } \frac{5500 \times 1 \times 10}{100} = \text{Rs } 550$$

Thus, the interest for two years is

Rs 500 + Rs 550 = Rs 1050.

We notice that the interest for the second year is more than that for the first year.

It is clear that in the second year, interest has been calculated on Rs 5500, which is equal to Rs 5,000 (Principal at the beginning) + Rs 500 (Interest for the first year). So, for the second year, interest on the interest has also been calculated. Interest calculated in this manner is known as compound interest.

Computation of Compound Interest by Using Formulae

Formula 1 :

Let P be the principal and the rate of interest be R% per annum. If the interest is compounded annually, then the amount A and the compound interest C.I. at the end of n years are given by

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$C.I. = A - P = P\left\{\left(1+\frac{R}{100}\right)^n - 1\right\}$$

Formula 2 :

Let P be the principal and the rate of interest be R% per annum. If the interest is compounded annually, then the amount A and the compound interest C.I. at the end of n years are given by

$$A = P\left(1+\frac{R}{100k}\right)^{nk}$$

and, C.I. = A – P

$$= P\left\{\left(1+\frac{R}{100k}\right)^{nk} - 1\right\}$$ respectively.

Here, interest is payable k times in a year.

Particular Cases :

Case 1 : When the interest is compound half-yearly or semi-annually.

In this case, k = 2

$$\therefore \quad A = P\left(1+\frac{R}{2\times 100}\right)^{2n}$$

and $$C.I. = P\left[\left(1+\frac{R}{2\times 100}\right)^{2n} - 1\right]$$

Case 2 : When interest is compounded quarterly.

In this case, k = 4

$$\therefore \quad A = P\left(1+\frac{R}{4\times 100}\right)^{4n}$$

and $$C.I. = P\left[\left(1+\frac{R}{4\times 100}\right)^{4} - 1\right]$$

❖ EXAMPLES ❖

Ex.23 A man deposits Rs 1,000 in a savings bank account. How much will it amount in three years if the rate of interest is 5% per annum and the interest is payable annually ?
(Solve without using formulae)

Sol. Interest on Rs 1000 for the first year

$$= \text{Rs } \frac{1000\times 1\times 5}{100}$$

= Rs 10 × 1 × 5

= Rs. 50

Amount after one year = Rs 1000 + Rs 50

= Rs 1050

$\therefore$ Interest for the second year = $\frac{1050\times 1\times 5}{100}$

$$= \text{Rs } \frac{105}{2} = \text{Rs } 52.50$$

Amount after two years = Rs 1050 + Rs 52.50

= Rs 1102.50

$\therefore$ Principal for the third year

$$= \text{Rs } \frac{1102.50\times 1\times 5}{100}$$

$$= \text{Rs } \frac{5512.5}{100}$$

= Rs 55.13

Amount after three years

= Rs 1102.50 + Rs 55.13

= Rs 1157.63

Thus, Rs 1,000 will become Rs 1,157.63 in three years.

Ex.24 Find the amount and the compound interest on Rs 5,000 lent at compound interest at 5% per annum for one year if the interest is payable half-yearly.
(Solve without using formulae)

Sol. Here, we calculate the compound interest for the period of one year in such a way that interest is calculated after six months. So, there will be two time intervals, each of six months, for the calculation of interest.

First Interval of Six Months

Interest on Rs 5000 for 6 months = $\frac{5000\times 5\times 1}{2\times 100}$

= Rs 125

∴ Amount at the end of the first interval of six months

= Rs 5000 + Rs 125 = Rs 5125

Second Interval of Six Months

Amount at the end of the first interval of six months will be taken as the principal for the second interval of six months.

interest on Rs 5125 for 6 months

$$= \text{Rs } \frac{5125 \times 5 \times 1}{2 \times 100} = \text{Rs } \frac{1025}{8}$$

= Rs 128.13

∴ Total interest on Rs 5000 for one year

= Rs 125 + Rs 128.13

= Rs 253.13.

Amount at the end of one year

= Rs 5000 + Rs 253.13

= Rs 5253.13.

Ex.25 Find the compound interest on Rs 90,000 for 3 years at the rate of 10% per annum compounded annually.

Sol. P = Rs 90000

n = 3 [∵ Interest is compounded annually]

r = 10% p.a.

Since $A = P\left(1+\frac{r}{100}\right)^n$

$$\therefore \quad A = \text{Rs } 90000\left(1+\frac{10}{100}\right)^3$$

$$= \text{Rs } 90000\left(1+\frac{1}{10}\right)^3$$

$$= \text{Rs } 90000\left(\frac{11}{10}\right)^3$$

$$= \text{Rs } 90000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10}$$

= Rs 90 × 11 × 11 × 11 = Rs 119790

Now, C.I. = A – P = Rs 119790 – Rs 90000

= Rs 29,790.

Ex.26 Calculate the amount due in 3 years on Rs 5,000, if the rates of compound interest for successive years are 7%, 8% and 10% respectively.

Sol. Interest for 1st year $= \text{Rs } \frac{5000 \times 7 \times 1}{100} = \text{Rs } 350$

and,amount at the end of 1st year

= Rs 5000 + Rs 350

= Rs 5350

= Principal for 2nd year

Interest for 2nd year $= \text{Rs } \frac{5350 \times 8 \times 1}{100}$

$= \text{Rs } \frac{42800}{100} = \text{Rs } 428$

and amount at the end of 2nd year

= Rs 5350 + Rs 428

= Rs 5778

= Principal for 3rd year

Interest for 3rd year $= \text{Rs } \frac{5778 \times 10 \times 1}{100}$

$= \text{Rs } \frac{57780}{100} = \text{Rs } 577.80$

and amount due at the end of 3rd year

= Rs 5778 + Rs 577.80

= Rs 6,355.80.

Ex.27 Compute the compound interest on Rs 20,000 for 2 years at 20% per annum when compounded half yearly.

Sol. Here,

Principal (P) = Rs 20000

Rate (r) = 20% per annum

$= \frac{20}{2}$ % or 10% per half year

Time(n) = 2 years

= 4 half years

Since, $A = P\left(1+\frac{r}{100}\right)^n$

$$\therefore \quad \text{Amount} = \text{Rs } 20000 \times \left(1+\frac{10}{100}\right)^4$$

$$= \text{Rs } 20000 \times \left(1+\frac{1}{10}\right)^4$$

$$= \text{Rs } 20000 \times \left(\frac{11}{10}\right)^4$$

$$= \text{Rs } 20000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10}$$

= Rs 2 × 11 × 11 × 11 × 11

= Rs 29282

∴ C.I. = A – P

∴ Compound Interest = Rs (29282 – 20000)

= Rs 9282.

Ex.28 Find the compound interest on Rs 15,625 for 9 months at 16% per annum, compounded quarterly.

Sol. Here, Principal (P) = Rs 15625

Rate (r) = 16% p.a.

= 4% per quarter

Time (n) = 9 months

= 3 quarters

Now, Amount, (A) $= P\left(1+\frac{r}{100}\right)^n$

$= \text{Rs } 15625\left(1+\frac{4}{100}\right)^3$

$= \text{Rs } 15625\left(1+\frac{1}{25}\right)^3$

$= \text{Rs } 15625\left(\frac{26}{25}\right)^3$

$= \text{Rs } 15625 \times \frac{26}{25} \times \frac{26}{25} \times \frac{26}{25}$

$= \text{Rs } 26 \times 26 \times 26$

$= \text{Rs } 17576$

Since, Compound Interest = Amount – Principal

$\therefore$ C.I. = Rs 17576 – Rs 15625

= Rs 1,951.

Inverse Problems -

[To find Principal, Time or Rate of Interest]

Ex.29 A certain sum was borrowed at 15% per annum. If at the end of 2 years, Rs 1,290 was compounded as C.I., then find the sum borrowed.

Sol. **First Method :**

Let the sum be Rs 100

Then, Amount $= \text{Rs } 100\left(1+\frac{15}{100}\right)^2$

$\left[\because \text{Amount} = P\left(1+\frac{r}{100}\right)^n\right]$

$= \text{Rs } 100\left(1+\frac{3}{20}\right)^2$

$= \text{Rs } 100\left(\frac{23}{20}\right)^2$

$= \text{Rs}\left(100 \times \frac{23}{20} \times \frac{23}{20}\right) = \text{Rs}\frac{529}{4}$

$\therefore$ Compound Interest $= \frac{529}{4} - 100 = \text{Rs}\frac{129}{4}$

If C.I. is Rs $\frac{129}{4}$, then the sum borrowed

= Rs 100

If C.I. is Rs 1,

then the sum borrowed = Rs $100 \times \frac{4}{129}$

If C.I. is Rs 1290, then the sum borrowed

$= \text{Rs } 100 \times \frac{4}{129} \times 1290$

$= \text{Rs } 100 \times 4 \times 10 = \text{Rs } 4{,}000$

Hence, the sum borrowed is Rs 4,000.

Alternate Method :

$\text{Amount} = P\left(1+\frac{r}{100}\right)^n = P\left(1+\frac{15}{100}\right)^2$

$= P\left(1+\frac{3}{20}\right)^2 = P\left(\frac{23}{20}\right)^2$

$= P \times \frac{23}{20} \times \frac{23}{20}$

$\therefore$ $\text{C.I.} = \left(P \times \frac{23}{20} \times \frac{23}{20}\right) - P$

$= P\left(\frac{529}{400} - 1\right)$

But, C.I. = Rs 1290

$\therefore$ $P\left(\frac{529}{400} - 1\right) = 1290$

or $P\left(\frac{529-400}{400}\right) = 1290$

or $P\left(\frac{129}{400}\right) = 1290$

or $P = \frac{129 \times 400}{129}$

or $P = 10 \times 400 = 4000$

Hence, Principal = Rs 4,000.

Ex.30 In how many years will Rs 800 amount to Rs 882 at 5% per annum compounded annually ?

Sol. Here, P = Rs 800

A = Rs 882

r = 5% p.a.

Let number of years be n.

Since, $A = P\left(1+\frac{r}{100}\right)^n$

$\therefore \quad 882 = 800\left(1+\frac{5}{100}\right)^n = 800\left(1+\frac{1}{20}\right)^n$

or $\frac{882}{800} = \left(1+\frac{1}{20}\right)^n$

or $\frac{441}{400} = \left(\frac{21}{20}\right)^n$

or $\left(\frac{21}{20}\right)^n = \left(\frac{21}{20}\right)^n$

$[\because 441 = 21^2 \text{ and } 400 = 20^2]$

Since the bases are same on both sides,

hence n = 2

Since interest is compounded annually

$\therefore$ Time = 2 years.

Ex.31 Determine the rate percent per annum if Rs 25,000 amounts to 26,010 in 6 months, interest being compounded quarterly.

Sol. Here, n = 2 $[\because$ 6 months = 2 quarters$]$

Now, $A = P\left(1+\frac{r}{100}\right)^n$, where r is the rate per quarter.

$\therefore \quad 26010 = 25000\left(1+\frac{r}{100}\right)^2$

or $\left(1+\frac{r}{100}\right)^2 = \frac{26010}{25000} = \frac{2601}{2500} = \left(\frac{51}{50}\right)^2$

or $\left(1+\frac{r}{100}\right) = \frac{51}{50}$

or $\frac{r}{100} = \frac{51}{50} - 1 = \frac{51-50}{50} = \frac{1}{50}$

or $r = \frac{1}{50} \times 100 = 2\%$

Hence, the required rate is 2% p.a.

➢ TIME AND WORK

We use the principles of direct and indirect variations to solve problems on 'time and work', such as :

"More men do more work and less men do less work" (Direct variation)

"More men take less time to do a work and less men take more time to do the same work."

(Indirect variation)

The problems on "time and work" are divided in two categories:

(i) To find the work done in a given period of time.

(ii) To find the time required to complete a given job.

Working Rules

We shall use the unitary method by considering the following fundamental rules for solving problems regarding time and work :

(i) A complete job or work is taken to be one.

(ii) Time to complete a work

$$= \frac{\text{Total work to be done}}{\text{Part of the work done in one day}}$$

❖ EXAMPLES ❖

Ex.32 Ratan takes 5 days to complete a certain job and shankar takes 8 days to do the same job. If both of them work together, how long will they take to complete the work ?

Sol. Since, Ratan takes 5 days to complete the given work

$\therefore$ Ratan finishes $\frac{1}{5}$ part in 1 day.

Similarly, Shankar takes 8 days to complete the work.

Therefore, Shankar finishes $\frac{1}{8}$ part in 1 day.

$\therefore$ In a day, they together will finish

$$= \frac{1}{5} + \frac{1}{8} = \frac{8+5}{40} = \frac{13}{40}$$

i.e., $\frac{13}{40}$ part of the work.

So, they both will take $\frac{40}{13}$ days $3\frac{1}{13}$ days to complete the work. Hence, the complete work will be finished by them together in $3\frac{1}{13}$ days.

Ex.33 Kshitij can do a piece of work in 20 days and Rohan can do the same work in 15 days. They work together for 5 days and then Rohan leaves. In how many days will Kshitij alone finish the remaining work ?

Sol. Since, Kshitij completes the work in 20 days

$\therefore$ Kshitij's 1 day work $= \frac{1}{20}$ part

Now, Rohan completes the work in 15 days.

Similarly, Rohan's 1 day work $= \frac{1}{15}$ part

$\therefore$ Their combined work for 1 day

$$= \frac{1}{20} + \frac{1}{15} = \frac{3+4}{60} = \frac{7}{60}$$

$\therefore$ Their combined work for 5 days

$$= 5 \times \frac{7}{60} = \frac{7}{12} \text{ part}$$

Remaining work

= Complete work – Work done in 5 days

$$= 1 - \frac{7}{12}$$

$$= \frac{12-7}{12} = \frac{5}{12} \text{ part}$$

Now, the remaining work is to be completed by Kshitij alone.

Kshitij can complete the whole work in 20 days.

So, he will complete $\frac{5}{12}$ work in $\left(\frac{5}{12} \times 20\right)$ days, i.e., $\frac{25}{3}$ days or $8\frac{1}{3}$ days.

Ex.34 A and B can do a piece of work in 10 days; B and C in 15 days; C and A in 12 days. How long would A and B take separately to do the same work ?

Sol. A and B can complete the work in 10 days.

$\therefore$ (A and B)'s one day work $= \frac{1}{10}$ part

Similarly,

(B and C)'s one day work $= \frac{1}{15}$ part

(C and A)'s one day work $= \frac{1}{12}$ part

Adding up, we get

2(A and B and C)'s work in 1 day

$$= \left(\frac{1}{10} + \frac{1}{15} + \frac{1}{12}\right) \text{part}$$

$$= \frac{6+4+5}{60} = \frac{15}{60} = \frac{1}{4} \text{ part}$$

$\therefore$ (A and B and C) can do in 1 day

$$= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} \text{ part}$$

Now,

Part of work A can do in 1 day

= (1 day work of A and B and C)

– (1 day work of B and C)

$$= \left(\frac{1}{8}\right) - \left(\frac{1}{15}\right)$$

$$= \frac{15-8}{120} = \frac{7}{120} \text{ part}$$

Hence, A can complete the work in $\left(1 \times \frac{120}{7}\right)$ days, i.e. $\frac{120}{7}$, or $17\frac{1}{7}$ days.

Similarly,

Part of the work B can do in 1 day

= (1 day work of A and B and C)

– (1 day work of A and C)

$$= \left(\frac{1}{8}\right) - \left(\frac{1}{12}\right) = \frac{3-2}{24} = \frac{1}{24}$$

Hence, B can complete the work in $\left(1 \times \frac{24}{1}\right)$ days, i.e., 24 days.

Ex.35 A contractor undertakes to construct a road in 20 days and engages 12 workers. After 16 days, he finds that only $\frac{2}{3}$ part of the work has been done. How many more workers should he now engage in order to finish the job in time ?

Sol. From the question, it is clear that $\frac{2}{3}$ part of the work has been completed by 12 workers in 16 days.

$\therefore$ Remaining work $= 1 - \frac{2}{3} = \frac{1}{3}$

Remaining number of days $= 20 - 16 = 4$

Thus, $\frac{1}{3}$ part of the work is to be finished in 4 days.

$\therefore$ Number of workers required to complete $\frac{2}{3}$ part of work in 16 days $= 12$

Number of workers required to complete 1 work in 16 days

$$= 12 \times \frac{3}{2} \times 16$$

Number of workers required to complete $\frac{1}{3}$ work in 1 day

$$= 12 \times \frac{3}{2} \times 16 \times \frac{1}{3}$$

Number of workers required to complete $\frac{1}{3}$ work in 4 days

$$= 12 \times \frac{3}{2} \times 16 \times \frac{1}{3} \times \frac{1}{4}$$

$\therefore$ Number of additional workers required

$= 24 - 12 = 12$

Hence, the contractor will have to engage 12 more workers to complete the work in time.