ALGEBRAIC EXPRESSIONS AND IDENTITIES

MULTIPLICATION OF ALGEBRAIC EXPRESSIONS

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We know that the product of two integers with the same sign is positive and the product of two integers with the opposite signs is negative.

i.e., $(+) \times (+) = (+)$, $(-) \times (-) = (+)$

$(+) \times (-) = (-)$, $(-) \times (+) = (-)$

Also, from the chapters of exponents, we know that

(i) $x^m \times x^n = x^{m+n}$

(ii) $(x^m)^n = x^{mn}$, where x, m, n are non-zero integers.

While multiplying algebraic expressions, we shall make use of these concepts.

Multiplying Two Monomials

Consider, $2x^3 \times 3x$

We known, $2x^3 = 2 \times x \times x \times x$ and $3x = 3 \times x$

So, $2x^3 \times 3x = (2 \times x \times x \times x) \times (3 \times x)$

$= (2 \times 3) \times (x \times x \times x \times x)$

$= 6x^4$

How did we perform the above multiplication ?

There are three steps :

(i) Multiply the coefficients of both the monomials.

(ii) Multiply the variables.

(iii) Multiply the above two results.

Ex.1 Multiply each of the following :

(i) $6xy$ and $5x^2y^2z$

(ii) $-7x^2yz$ and $\frac{2}{3}xy^3$

(iii) $\left(\frac{-8}{5}a^2bc^3\right)$ and $\left(\frac{-3}{4}ab^2x\right)$

Sol. (i) $6xy \times 5x^2y^2z = (6 \times 5) \times (x \times x^2) \times (y \times y^2) \times z$

$= (6 \times 5) \times x^{1+2} \times y^{1+2} \times z$ (Using $x^m \times x^n = x^{m+n}$)

$= 30x^3y^3z$

(ii) $\left(-7x^2yz\right)\times\left(\frac{2}{3}xy^3\right) = \left(-7\times\frac{2}{3}\right)\left(x^2\times x\right)\times\left(y\times y^3\right)\times z$

$= \frac{-14}{3}x^{2+1}y^{1+3}z$ $\left(\text{Using } x^m \times x^n = x^{m+n}\right)$

$= \frac{-14}{3}x^3y^4z$

(iii) $\left(\frac{-8}{5}a^2bc^3\right)\times\left(\frac{-3}{4}ab^2x\right) = \left(\frac{-8}{5}\times\frac{-3}{4}\right)\times\left(a^2\times a\right)\times\left(b\times b^2\right)\times c^3\times x$

$= \frac{6}{5}a^{2+1}b^{1+2}c^3x = \frac{6}{5}a^3b^3c^3xc$

Multiplying Three or More Monomials

The following rule can be used for multiplying any number of monomials.

(i) The coefficient of the product of the given monomials is the product of the coefficients of these monomials.

(ii) The exponent of each literal is the sum of the exponents of this literal in the given monomials.

Ex.2 Find the product of

(i) $5xyz$, $10x^2y^2z^2$, $-3x^2y^3z^4$ and $6x^2y^2z^5$

(ii) $-8xyz$, $4x^3y^2z^2$, $3x^2y^2z^2$ and $-2yz$

Sol. (i) $5xyz \times 10x^2y^2z^2 \times (-3x^2y^3z^4) \times 6x^2y^2z^5$

$= 5 \times 10 \times (-3) \times 6 \times (x \times x^2 \times x^2 \times x^2) \times (y \times y^2 \times y^3 \times y^2) \times$

$(z \times z^2 \times z^4 \times z^5)$

$= -900 \times x^{1+2+2+2}\, y^{1+2+3+2}\, z^{1+2+4+5}$

$= -900\, x^7\, y^8\, z^{12}$

(ii) $(-8xyz) \times 4x^3y^2z^2 \times 3x^2y^2z^2 \times (-2yz)$

$= -8 \times 4 \times 3 \times (-2) \times (x \times x^3 \times x^2) \times (y \times y^2 \times y^2 \times y) \times (z \times z^2 \times z^2 \times z)$

$= 192 \times x^{1+3+2} \times y^{1+2+2+1} \times z^{1+2+2+1}$

$= 192\, x^6y^6z^6$

Ex.3 Obtain the volume of the rectangular box whose length, breadth and height are xy, $2x^2y$ and $2xy^2$ respectively.

Sol We know that volume of a rectangular box is the product of its length, breadth and height.

Volume of the box $= xy \times 2x^2y \times 2xy^2$

$= 2 \times 2 \times (x \times x^2 \times x) \times (y \times y \times y^2)$

$= 4 \times x^4 \times y^4 = 4x^4y^4$.

Ex.4 Find the value of $5a^6 \times (-10ab^2) \times \left(-\frac{1}{25}a^2b^3\right)$ for $a = 1$ and $b = 2$.

Sol We have,

$5a^6 \times (-10ab^2) \times \left(-\frac{1}{25}a^2b^3\right)$

$= 5 \times (-10) \times \left(-\frac{1}{25}\right) \times (a^6 \times a \times a^2) \times (b^2 \times b^3)$

$= 2 \times a^{6+1+2} \times b^{2+3} = 2a^9b^5$

Putting $a = 1$ and $b = 2$, we have

$2a^9b^5 = 2 \times (1)^9 \times (2)^5 = 2 \times 1 \times 32 = 64$

Multiplying a Monomial by a Bionomial

In the case of integers we use the distributive property of multiplication over addition for simpifying the products like $x \times (y + z)$. Similarly, in case of algebraic expressions, we use this property. If A, B and C are three monomials, then $A \times (B + C) = A \times B + A \times C$

Ex.5 Find the product of $9x^2y$ and $(x + 2y)$.

Sol $9x^2y \times (x + 2y) = (9x^2y \times x) + (9x^2y \times 2y)$

$= 9x^3y + 9 \times 2 \times (x^2y \times y)$

$= 9x^3y + 18x^2y^2$

The method of multiplying a monomial and a binomial discussed above is called the horizontal or row method, because the working is done horizontally. We can also multiply a monomial and a binomial vertically i.e., from top to bottom in columns, which is called vertical method. Observe the following example :

e.g. Multiply 9xy and $3xy + 5y^2$.

Sol.

$$\begin{array}{r} 3xy + 5y^2 \\ \times \quad 9xy \\ \hline 27x^2y^2 + 45xy^3 \\ \hline \end{array}$$

[Multiply 3xy and $5y^2$ by 9xy separately and add them]

$9xy \times 3xy$ $\quad$ $9xy \times 5y^2$

Multiplying a Monomial by a Trinomial

If A, B, C and D are four monomials, then $A \times (B + C + D) = A \times B + A \times C + A \times D$

Ex.6 Multiply 3l and l – 4m + 5n

Sol $3l\,(l - 4m + 5n) = 3l \times l - 3l \times 4m + 3l \times 5n$

$= 3l^2 - 12lm + 15\,ln$

Multiplying a Binomial by a Binomial

Multiplication of two binomials can be performed using distributive property of multiplication over addition twice. If A, B, C and D are four monomials, then

$(A + B) \times (C + D) = A \times (C + D) + B \times (C + D)$

$= A \times C + A \times D + B \times C + B \times D$

$= AC + AD + BC + BD.$

This multiplication can also be performed by column method, as follows :

$$\begin{array}{r} A+B \\ \times\ C+D \\ \hline AD+BD \\ AC+BC \\ \hline AC+AD+BC+BD \\ \hline \end{array} \qquad \begin{array}{l} \\ \\ [\text{Multiply } (A+B) \text{ by } D] \\ [\text{Multiply } (A+B) \text{ by } C] \end{array}$$

Multiplying a Binomial by a Trinomial

Multiplication of a binomial and a trinomial can be performed using distributive property of multiplication over addition. If A, B, C, D and E are five monomials, then

$(A + B) \times (C + D + E) = A \times (C + D + E) + B \times (C + D + E)$

$= AC + AD + AE + BC + BD + BE.$

This multiplication can also be performed by column method, al follows :

$$\begin{array}{r} C+D+E \\ \times\ A+B \\ \hline BC+BD+BE \\ AC+AD+AE \\ \hline AC+AD+AE+BC+BD+BE \\ \hline \end{array}$$

Ex.7 Multiply the following :

(i) $(9a + 6b)$ and $(6a + 9b)$

(ii) $(2x^2y + 3y^2)$ and $(8y - 4x^2)$

Sol (i) Using distributive property of multiplication over addition :

$(9a + 6b) \times (6a + 9b) = 9a \times (6a + 9b) + 6b \times (6a + 9b)$

$= 9a \times 6a + 9a \times 9b + 6b \times (6a + 9b)$

$= 54a^2 + 81ab + 36ab + 54b^2$

$= 54a^2 + 117ab + 54b^2$

By column method :

$$\begin{array}{r} 9a+6b \\ \times\ 6a+9b \\ \hline 81ab+54b^2 \\ 54a^2+36ab \\ \hline 54a^2+117ab+54b^2 \\ \hline \end{array}$$

(ii) Using distributive property of multiplication over addition :

$(2x^2y + 3y^2) \times (8y - 4x^2)$

$= 2x^2y \times (8y - 4x^2) + 3y^2 \times (8y - 4x^2)$

$= 2x^2y \times 8y + 2x^2y \times (-4x^2) + 3y^2 \times 8y + 3y^2 \times (-4x^2)$

$= 16x^2y^2 - 8x^4y + 24y^3 - 12x^2y^2$

$= 16x^2y^2 - 12x^2y^2 - 8x^4y + 24y^3$

$= 4x^2y^2 - 8x^4y + 24y^3$