ALGEBRAIC EXPRESSIONS AND IDENTITIES

INTRODUCTION OF ALGEBRAIC EXPRESSION

INTRODUCTION

Algebra is the branch of mathematics concerning the study of the rules of operations and relations. Elementary algebra is the most basic form of algebra. It is taught to students who are presumed to have no knowledge of mathematics beyond the basic principles of arithmetic. In arithmetic, only numbers and their arithmetical operations (such as +, -, \times , ,) occur. In algebra, numbers are often denoted by symbols (such as a, x or y). This is useful because :

A polynomial is an expression that is constructed from one or more variables and constants, using only the operations of addition, subtraction, and multiplication.

for example, $x^2 + 2x - 3$ is a polynomial in the single variable x.

An important class of problems in algebra is factorization of polynomials, that is expressing a given polynomial as a product of other polynomials. The above polynomial can be factored as

(x - 1) (x + 3).

ALGEBRAIC EXPRESSIONS

The branch of mathematics which deals with numbers is called Arithmetic. Algebra can be considered as generalisation of arithmetic, where we use letter in place of numbers.

Constants:

A symbol having a fixed numerical value is called a constant.

For example, 8, -6, 5/7, p etc are all constants

Variables:

A symbol which may be assinged different numerical values is known as a variable. For example, circumference of a circle is given byc = 2pr Here, 2 and p are constants, while c and r are variables.

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Terms of an Algebraic Expression:

The several parts of an algebraic expression separated by + or – operations are called the terms of the expression.

For example: $4 + 9x - 5x^2y + \frac{3}{5}xy$ is an algebraic expression containing four terms, namely,

4, 9*x*,
$$-5x^2y$$
 and $\frac{3}{5}xy$.

Factors of term :

Ex. The term $9y^2$ is a product of 9, y and y. Thus 9, y and y are the factors of $9y^2$.

Coefficient of a term :

Consider a algebraic expression $3x^2 + 5x + 6$. In $3x^2 + 5x + 6$, $3x^2$ is first term, 5x is second term and 6 is the third term. In the first term $3x^2$, 3 is called numerical coefficient and x^2 is called literal coefficient. Similarly in the second term 5x, 5 is called numerical coefficient and x is called literal coefficient.

Like terms :

In any algebraic expression, the terms having the same literal coefficients are called like terms.

For example: $6x^3$, $-x^3$, $2x^3$ and $\frac{1}{4}x^3$ are like terms.

Unlike terms :

In any algebraic expression, the terms having different literal coefficient are called unlike terms.

For example : 7x, x^2 , $2x^3$ and $15x^4$ are unlike terms.

Algebraic Expressions:

A combination of constants and variables, connected by operations +, -, \times and $_{,}$ is known as an algebraic expressions.

Type of Algebraic Expressions

Monomial - one term. Ex. : $3p^2q^2$ **Binomial** - two terms. Ex. : 3x + 4y

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Trinomial - three terms. Ex. : $x^2 + y^2 + 6$

POLYNOMIALS

An algebraic expression f(x) of the form $f(x) = a_0 + a_1x + a_2x^2 + \dots + a^nx^n$; where a_0 , a_1, a_2, \dots, a_n are real numbers and all the indices of variable x are non-negative integers, is called a polynomial in variable x and the highest indices n is called the degree of the polynomial, if $a_n^{-1} 0$. Here, a_0, a_1x, a_2x^2 and a_nx^n are called the terms of the polynomial and a_0, a_1, a_2 a_n are called various coefficients of the polynomial f(x). A polynomial in x is said to be in standard form when the terms are written either in increasing order or in decreasing order of the indices of x in various terms.

For example: $x^2 - a^2$, $ax^2 + bx + c$, $x^3 + 3x^2 + 3x + 1$, $y^3 - 7y + 6$ etc. are the polynomials written in their standard form.

Simplest form of Polynomial:

A polynomial is said to be in simplest form when no two of the polynomial are like terms.

For example: $3x^2 + 4x + 2$.

Standard form of Polynomial :

When a polynomial is written in either ascending or descending power of variable.

For example: $x^3 - 2x^2 + 3x - 6$

Degree of a Polynomial in One Variable:

In case of a polynomial in one variable, the highest power of the variable is called the degree of the polynomial.

Degree of a Polynomial in Two or More Variables:

In case of polynomials in more than one variable, the sum of the powers of the variables in each term is taken up and the highest sum so obtained is called the degree of the polynomial.

Types of polynomials

Polynomials can be classified on the basis of number of terms and on the basis of degree.

On the basis of degree :

(i) Zero polynomial :

A polynomial f(x) = 0 is called zero polynomial. Its degree is not defined.

(ii) Constant polynomial :

A polynomial of degree zero is called a constant polynomial.

For example : 2, -5, 7. Every real number is a constant polynomial

(iii) Linear polynomial:

A polynomial of degree 1 is called a linear polynomial.

For example : 9x + 5 is a linear polynomial in *x*.

x + y + 4 is a linear polynomial in x and y.

(iv) Quadratic polynomial:

A polynomial of degree 2 is called a quadratic polynomial.

For example : $2y^2 - 8y + 5$ is a quadratic polynomial in y.

2xy+5x+3y+4 is a quadratic polynomial in x and y.

(v) Cubic polynomial :

A polynomial of degree 3 is called a cubic polynomial.

For example : $2x^3 - 3x^2 + 5x + 1$ is a cubic polynomial in *x*

 $2x^2y + 5xy^2 + 8$ is a cubic polynomial in *x* and *y*.

(vi) Biquadratic polynomial:

A polynomial of degree 4 is called a biquadratic polynomial.

For example : $z^4 + 6z^3 + 10z^2 + 6z + 1$ is biquadratic polynomial in z.

 $3x^2yz + 4xy^2z + 5xyz^2$ is biquadratic polynomial in, x, y and z.

Ex.1 Find the degree of each of the following polynomials.

(i)
$$2x^3 + x^2 - x + 4$$
 (ii) $x + 4 - 3x^3 + x^4$ (iii) 10

Sol. (i) $2x^3 + x^2 - x + 4$:

The highest power term is $2x^3$. The power of variable in this term is 3. So the degree of given polynomial is 3.

(ii) $x + 4 - 3x^3 + x^4$:

The highest power term is x^4 .

Degree of polynomial is 4.

(iii) 10:

10 is a constant polynomial. It can be written as 10. x^0 (As $x^0 = 1$) where x is any variable. Highest power of variable is 0 (zero), so the degree of constant polynomial is '0'.

'0' itself is a constant polynomial.

Ex.2 Classify the following polynomials as linear, quadratic or cubic polynomials.

(i) $10x^2$ (ii) y (iii) 1 + z(iv) $y + y^3$ (v) $x^2 + x + 5$

- **Sol.** (i) $10x^2$: Degree of polynomial $10x^2$ is '2' so it is a quadratic polynomial.
 - (ii) y : Degree of polynomial is '1' so the polynomial is a quadratic polynomial.
 - (iii) 1 + z: Degree of polynomial is '1'. It is a linear polynomial.

(iv) $y + y^3$: Degree of polynomial is '3'. It is a cubic polynomial.

(v) $x^2 + x + 5$: Degree of polynomial is '2'. It is a quadratic polynomial.