ALGEBRAIC EXPRESSIONS AND IDENTITIES

IDENTITIES

STANDARD IDENTITIES

Let us learn some useful identities which involve the product of two binomials.

Identity 1

$(a + b) \times (a + b)$

$(a + b)(a + b) = a(a + b) + b(a + b)$

$= a \times a + a \times b + b \times a + b \times b$

$= a^2 + ab + ba + b^2$

$= a^2 + 2ab + b^2$

$(a + b)^2 = a^2 + 2ab + b^2$

Identity 2

$(a - b) \times (a - b)$

$(a - b)(a - b) = a \times (a - b) - b \times (a - b)$

$= a \times a + a \times (-b) + (-b) \times a + (-b) \times (-b)$

$= a^2 - ab - ba + b^2$

$= a^2 - 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

Identity 3

$(a - b) \times (a + b)$

$(a - b)(a + b) = a \times (a + b) - b \times (a + b)$

$= a \times a + a \times b + (-b) \times a + (-b) \times b$

$= a^2 + ab - ba - b^2$

$= a^2 - b^2$

$(a + b)(a - b) = a^2 - b^2$

Identity 4

$(x + a)(x + b) = x \times (x + b) + a \times (a + b)$

$= x \times x + x \times b + a \times x + a \times b$

$= x^2 + xb + ax + ab$

$= x^2 + (b + a)x + ab$

$= x^2 + (a + b)x + ab$

$(x + a)(x + b) = x^2 + (a + b)x + ab$

Let us now take up an example to find the importance to these identities.

Ex.1 Expand $\left(\frac{1}{2}x+\frac{3}{7}y\right)^2$

Sol. $\left(\frac{1}{2}x+\frac{3}{7}y\right)\left(\frac{1}{2}x+\frac{3}{7}y\right)=\frac{1}{2}\times x\left(\frac{1}{2}x+\frac{3}{7}y\right)+\frac{3}{7}y\left(\frac{1}{2}x+\frac{3}{7}y\right)$

$=\frac{1}{2}x\times\frac{1}{2}x+\frac{1}{2}x\times\frac{3}{7}y+\frac{3}{7}y\times\frac{1}{2}x+\frac{3}{7}y\times\frac{3}{7}y$

$=\frac{1}{4}x^2+\frac{3}{14}xy+\frac{3}{14}xy+\frac{9}{49}y^2$

$=\frac{1}{4}x^2+\left(\frac{3}{14}+\frac{3}{14}\right)xy+\frac{9}{49}y^2$

$=\frac{1}{4}x^2+\frac{6}{14}xy+\frac{9}{49}y^2=\frac{1}{4}x^2+\frac{3}{7}xy+\frac{9}{49}y^2$

Let us solve this by using identity,

$\left(\frac{1}{2}x+\frac{3}{7}y\right)^2=\left(\frac{1}{2}x\right)^2+2\times\frac{1}{2}x\times\frac{3}{7}y+\left(\frac{3}{7}y\right)^2$

$=\frac{1}{4}x^2+\frac{3}{7}xy+\frac{9}{49}y^2$

Thus, we see it is much simpler to use the identity rather than multiplying the binomial by itself.

If we do direct multiplication, it involves more steps and more tedious calculations.

It is easier to use identities.

Ex.2 Evaluate :

(i) $(2x + 3y)^2$

(ii) $(2x - 3y)^2$

(iii) $(2x + 3y)(2x - 3y)$

Sol. (i) We have,

$(2x + 3y)^2 = (2x)^2 + 2 \times (2x) \times (3y) + (3y)^2$

[Using: $(a + b)^2 = a^2 + 2ab + b^2$]

$= 4x^2 + 12xy + 9y^2$

(ii) We have,

$(2x - 3y)^2 = (2x)^2 - 2 \times (2x) \times (3y) + (3y)^2$

[Using: $(a - b)^2 = a^2 - 2ab + b^2$]

$= 4x^2 - 12xy + 9y^2$

(iii) We have,

$(2x + 3y)(2x - 3y) = (2x)^2 - (3y)^2$

[Using : $(a + b)(a - b) = a^2 - b^2$]

$= 4x^2 - 9y^2$.

Ex.3 Write down the squares of each of the following binomials :

(i) $\left(x+\frac{a}{2}\right)$ (ii) $\left(5b-\frac{1}{2}\right)$ (iii) $\left(y+\frac{y^2}{2}\right)$

Sol. (i) We have,

$\left(x+\frac{a}{2}\right)^2 = x^2 + 2 \times x \times \frac{a}{2} + \left(\frac{a}{2}\right)^2$

[Using : $(a + b)^2 = a^2 + 2ab + b^2$]

$= x^2 + xa + \frac{a^2}{4}$

(ii) We have,

$$\left(5b-\frac{1}{2}\right)^2 = (5b)^2 - 2 \times 5b \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$$

[Using : $(a - b)^2 = a^2 - 2ab + b^2$]

$$= 25b^2 - 5b + \frac{1}{4}$$

(iii) We have,

$$\left(y+\frac{y^2}{2}\right)^2 = y^2 + 2 \times y \times \frac{y^2}{2} + \left(\frac{y^2}{2}\right)^2$$

$$= y^2 + y^3 + \frac{y^4}{4}$$

Ex.4 If $x + \frac{1}{x} = 4$, find the values of

(i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$

Sol. (i) We have,

$$x + \frac{1}{x} = 4$$

On squaring both sides, we get

$$\left(x+\frac{1}{x}\right)^2 = 4^2$$

$$\Rightarrow x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 16$$

$$\Rightarrow x^2 + 2 + \frac{1}{x^2} = 16$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 16 - 2$$

[On transposing 2 on RHS]

$$\Rightarrow x^2 + \frac{1}{x^2} = 14$$

(ii) We have,

$$x^2 + \frac{1}{x^2} = 14$$

On squaring both sides, we get

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 14^2$$

$$\Rightarrow (x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2 \times x^2 \times \frac{1}{x^2} = 196$$

$$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 196$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 196 - 2$$

[On transposing 2 on RHS]

$$\Rightarrow x^4 + \frac{1}{x^4} = 194$$