COMPARING QUANTITIES

RATIOS AND PERCENTAGES

INTRODUCTION (RATIO)

A ratio is a comparison of two numbers (quantities) by division. The ratio of a to b is written as a : b or .

In the ratio a : b, a and b are called terms of the ratio. 'a' is the antecedent and 'b' is the consequent.

A ratio is a number, so to find the ratio of two quantities they must be expressed in the same units.

Ex.1 Find the (i) ratio of M20 to M80 (ii) ratio of 3 km to 600m

Sol (i) 20 : 80 or 1 : 4

(ii) 3000m : 600m or 3000 : 600 or 5 : 1

Properties of Ratios

(a) In a ratio, two quantities are compared. So, the quantities must be of the same kind. i.e.,
they must be expressed in the same units.

(b) The value of a ratio remains unaltered if the given ratio is multiplied or divided by the same non-zero quantity. If a, b and m are non-zero real numbers then a : b = ma : mb and a : b = a/m : b/m

Dividing a Given Number in the Given Ratio

Let 'X' be the given number, the given ratio is $x_1 : x_2$. Now, x is to be divided in the ratio $x_1 : x_2$.

X is to be divided into two parts such that

Value of first part: Value of second part = $x_1 : x_2$

Therefore

First part $= \left(\frac{x_1}{x_1 + x_2} \right) \times X$

Second part $= \left(\frac{x_2}{x_1 + x_2} \right) \times X$

Ex.1 Two numbers are in the ratio 4:5. If the sum of the number is 63, then the numbers.

Sol. Here 63 is to be divided in the ratio 4:5

First part (number) $= \frac{4}{4+5} \times 63 = \frac{4}{9} \times 63 = 28$

Second part (number) $= \frac{5}{4+5} \times 63 = \frac{5}{9} \times 63 = 35$

Or second part = 63 – 28 = 35.

Certain Types of Ratios

(a) Compounded Ratio: The compounded ratio of a : b and c : d is ac : bd.

(b) Duplicate Ratio : Duplicate ratio of a : b is $a^2 : b^2$

(c) Triplicate Ratio : The triplicate ratio of a : b is $a^3 : b^3$

(d) Sub-duplicate Ratio : The Sub-duplicate Ratio of a : b is $\sqrt{a} : \sqrt{b}$.

(e) Sub – triplicate Ratio : The Sub – triplicate Ratio of a : b is $\sqrt[3]{a} : \sqrt[3]{b}$.

(f) Inverse Ratio : The Inverse Ratio or reciprocal ratio of a : b is $\frac{1}{a} : \frac{1}{b}$ i.e. b : a

Proportion

Equality of two ratios is called proportion.

If a : b = c : d, then a, b, c and d are in proportional, a : b = c : d is also represented as a : b :: c : d.

(i) The first and the fourth (last) term are called extremes and the second and third terms are called middle terms or means.

(ii) In a proportion a : b :: c : d.

Product of extremes = product of means.

For example : 1 : 4 :: 3 : 12 is a proportional here

we see $1 \times 12 = 4 \times 3 \quad \Rightarrow 12 = 12.$

(iii) If a : b :: c : d then b : a :: d : c or $\frac{a}{b}=\frac{c}{d}\Leftrightarrow\frac{b}{a}=\frac{d}{c}$

This is known as Invertendo.

(iv) If a : b :: c : d then a : c :: b : d or $\frac{a}{b}=\frac{c}{d}\Leftrightarrow\frac{a}{c}=\frac{b}{d}$

This result is known as Alternendo.

(v) If $a:b::c:d$

$\frac{a}{b}+1=\frac{c}{d}+1$

$\frac{a+b}{b}=\frac{c+d}{d}$

or $a:b::c:d$

$(a+b):b::(c+d):d$

This result is fonown as Componendo

(vi) If $a:b::c:d$

$\frac{a}{b}-1=\frac{c}{d}-1$

$\frac{a-b}{b}=\frac{c-d}{d}$

or $a:b::c:d$

$(a-b):b::(c-d):d$

This result is known as Diridendo.

(vii) Dividing equation (A) and (B) we get

$$\frac{\frac{a+b}{b}}{\frac{a-b}{b}}=\frac{\frac{c+d}{d}}{\frac{c-d}{d}}$$

$(a+b):(a-b):(c+d)(c-d)$

or $a:b:ac:d$

$(a+b):(a-b):(c+d)(c-d)$

This result is known as Componendo and Dividendo

(viii) a, b, c, d are said to be in continued proportion

If $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}$

PERCENTAGE

INTRODUCTION (PERCENTAGE)

The word percent means per hundred or for every hundred. The symbol % is used for the term percent. Thus 20 percent written as 20% and it means 20 out of 100. This can also be written as $\frac{20}{100}$.

Interpretation of Percentage

In a class of 50 students, 40% are girls. How many girls are there ?

The given statement means that,

If the class strength is 100, out of them 40 are girls.

If the class strength is 50, number of girls $= \frac{50\times 40}{100} = 20$

Rahul says his salary is M20,000 per month. He saves 35% of it. How much does he save ?

He means for every M100, he saves M35.

For M20,000, he saves $= \frac{20,000\times 35}{100} =$ M7000.

Conversion of percent into fraction :

To convert a percent into fraction, divide it by 100 and remove the % sign.

Ex.2 Convert 15% into fraction.

Sol. $15\% = \frac{15}{100}$

Conversion of fraction into percent

To convert any fraction to percent, multiply it by 100 and put the % sign.

Ex. 3 Convert $\frac{2}{5}$ into percent.

Sol $\frac{2}{5} = \frac{2}{5} \times 100\% = 40\%$

Conversion of percent into ratio

First change the percent into fraction by dividing it by 100 and remove the % sign, finally reduce the obtained fraction into its simplest form.

Ex.4 Convert 35%, 0.65% and 4.5% into ratio.

Sol $35\% = \frac{35}{100} = \frac{7}{20} = 7:20$

$0.65\% = \frac{0.65}{100} = \frac{65}{10000} = \frac{13}{2000} = 13:2000$

$4.5\% = \frac{4.5}{100} = \frac{45}{1000} = \frac{9}{200} = 9:200$

Conversion of ratio into percent

$3:4 = \left(\frac{3}{4} \times 100\right)\% = 75\%$

Conversion of percent into decimal

To convert the percent to decimal, first change it to fraction by dividing it by 100 and remove the %sign, finally put the decimal point accordingly.

Ex.5 Convert 18% to decimal.

Sol $18\% = \frac{18}{100} = 0.18$

Conversion of decimal to percent

First change the decimal into fraction and then multiply by 100 and put the % sign.

Ex.6 Convert 1.5 and 0.25 to percent.

Sol $1.5 = \frac{15}{10} = \left(\frac{15}{10} \times 100\right)\% = 150\%$

$0.25 = \frac{25}{100} = \left(\frac{25}{100} \times 100\right)\% = 25\%$

Percentage of a number

To find percentage of a given number, multiply the given number by required percentage.

Ex.7 Find 20% of 400.

Sol Let the required value is x.

$x = 20\%$ of 400

$x = \frac{20}{100} \times 400 = 80$

20% of 400 is 80.

Original number from its percentage

Ex.8 Find the number whose 20% is 60.

Sol Let the required number is x.

Now, 20 % of x is 60.

20% of $x = 60$ $\frac{20}{100} \times x = 60$

$x = \frac{60 \times 100}{20} = 300$ The required number is 300.

Percentage of one quantity to another quantity

If x% of a is b, then $\frac{x}{100} \times a = b$

$x = \frac{100\ b}{a}$

Ex.9 What percent of 120 is 30.

Sol Let x% of $120 = 30$.

$\frac{x}{100} \times 120 = 30$

$x = \frac{30 \times 100}{120} = 25$

25% of 120 is 30.