COMPARING QUANTITIES

COMPOUND INTEREST

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When we borrow money from a financial agency (bank, financial agency or individual), it is called the lender.

The borrowed money is called the principal.

We have to pay some additional money together with the borrowed money for a certain time period, for the benefit of using his or her money. The additional money that we pay is called the interest.

If the principal remains the same for the whole loan period (or time), then the interest is called the simple interest.

The interest together with the principal is called the amount.

If the principal does not remain the same for the whole loan period due to addition of (compounding of) interest to the principal after a certain interval of time to form the new principal, then the interest so obtained is called the compound interest.

Simple Interest :

(i) Simple Interest $= \frac{\text{Principal} \times \text{Rate of interest} \times \text{Time}}{100}$

(ii) Amount = Principal + Simple Interest.

Compound Interest :

To understand compound interest, we consider the following example -

"A man lends Rs 5,000 to a finance company at 10% per annum. What interest does he get after one year ? What will be the amount then ? At the end of the year, if he decides to deposit the whole sum (amount after one year) for another year, what interest does he get at the end of the second year ?"

Interest after one year $= \text{Rs } \frac{5000 \times 1 \times 10}{100} = \text{Rs } 500$

$\therefore$ Amount after one year $= \text{Rs } 5000 + \text{Rs } 500 = \text{Rs } 5500.$

When the deposit is Rs 5,500 in the company for one more year, the amount of Rs 5,500 due at the end of first year becomes the principal for the second year.

$\therefore$ Interest at the end of the second year

$= \text{Rs } \frac{5500 \times 1 \times 10}{100} = \text{Rs } 550$

Thus, the interest for two years is

$\text{Rs } 500 + \text{Rs } 550 = \text{Rs } 1050.$

We notice that the interest for the second year is more than that for the first year.

It is clear that in the second year, interest has been calculated on Rs 5500, which is equal to Rs 5,000 (Principal at the beginning) + Rs 500 (Interest for the first year). So, for the second year, interest on the interest has also been calculated. Interest calculated in this manner is known as compound interest.

Computation of Compound Interest by Using Formulae

Formula 1 :

Let P be the principal and the rate of interest be R% per annum. If the interest is compounded annually, then the amount A and the compound interest C.I. at the end of n years are given by

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$C.I. = A - P = P\left\{\left(1+\frac{R}{100}\right)^n - 1\right\}$$

Formula 2 :

Let P be the principal and the rate of interest be R% per annum. If the interest is compounded annually, then the amount A and the compound interest C.I. at the end of n years are given by

$$A = P\left(1+\frac{R}{100k}\right)^{nk}$$

and, C.I. = A – P

$$= P\left\{\left(1+\frac{R}{100k}\right)^{nk} - 1\right\}$$ respectively.

Here, interest is payable k times in a year.

Particular Cases :

Case 1 : When the interest is compound half- yearly or semi-annually.

In this case, k = 2

$$\therefore \quad A = P\left(1+\frac{R}{2\times 100}\right)^{2n}$$

And C.I. $= P\left[\left(1+\frac{R}{2\times100}\right)^{2n}-1\right]$

Case 2 : When interest is compounded quarterly.

In this case, $k = 4$

$\therefore \quad A = P\left(1+\frac{R}{4\times100}\right)^{4n}$

And C.I. $= P\left[\left(1+\frac{R}{4\times100}\right)^{4}-1\right]$

Ex.1 A man deposits Rs 1,000 in a savings bank account. How much will it amount in three years if the rate of interest is 5% per annum and the interest is payable annually ? (Solve without using formulae)

Sol. Interest on Rs 1000 for the first year

$= \text{Rs } \frac{1000\times1\times5}{100}$

$= \text{Rs } 10 \times 1 \times 5$

$= \text{Rs. } 50$

Amount after one year $= \text{Rs } 1000 + \text{Rs } 50 = \text{Rs } 1050$

$\therefore$ Interest for the second year $= \frac{1050\times1\times5}{100}$

$= \text{Rs } \frac{105}{2} = \text{Rs } 52.50$

Amount after two years $= \text{Rs } 1050 + \text{Rs } 52.50 = \text{Rs } 1102.50$

$\therefore$ Principal for the third year

$= \text{Rs } \frac{1102.50\times1\times5}{100}$

$= \text{Rs } \frac{5512.5}{100}$

$= \text{Rs } 55.13$

Amount after three years

$= \text{Rs } 1102.50 + \text{Rs } 55.13$

$= \text{Rs } 1157.63$

Thus, Rs 1,000 will become Rs 1,157.63 in three years.

Ex.2 Find the amount and the compound interest on Rs 5,000 lent at compound interest at 5% per annum for one year if the interest is payable half-yearly.

(Solve without using formulae)

Sol. Here, we calculate the compound interest for the period of one year in such a way that interest is calculated after six months. So, there will be two time intervals, each of six months, for the calculation of interest.

First Interval of Six Months

Interest on Rs 5000 for 6 months $= \frac{5000 \times 5 \times 1}{2 \times 100} = \text{Rs } 125$

$\therefore$ Amount at the end of the first interval of six months

$= \text{Rs } 5000 + \text{Rs } 125 = \text{Rs } 5125$

Second Interval of Six Months

Amount at the end of the first interval of six months will be taken as the principal for the second interval of six months.

interest on Rs 5125 for 6 months

$= \text{Rs } \frac{5125 \times 5 \times 1}{2 \times 100} = \text{Rs } \frac{1025}{8}$

$= \text{Rs } 128.13$

$\therefore$ Total interest on Rs 5000 for one year

= Rs 125 + Rs 128.13

= Rs 253.13.

Amount at the end of one year

= Rs 5000 + Rs 253.13

= Rs 5253.13.

Ex.3 Find the compound interest on Rs 90,000 for 3 years at the rate of 10% per annum compounded annually.

Sol. P = Rs 90000

n = 3 [$\because$ Interest is compounded annually]

r = 10% p.a.

Since $A = P\left(1+\frac{r}{100}\right)^n$

$\therefore$ $A = \text{Rs } 90000\left(1+\frac{10}{100}\right)^3$

$= \text{Rs } 90000\left(1+\frac{1}{10}\right)^3$

$= \text{Rs } 90000\left(\frac{11}{10}\right)^3$

$= \text{Rs } 90000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10}$

$= \text{Rs } 90 \times 11 \times 11 \times 11 = \text{Rs } 119790$

Now, C.I. = A – P = Rs 119790 – Rs 90000

= Rs 29,790.

Ex.4 Calculate the amount due in 3 years on Rs 5,000, if the rates of compound interest for successive years are 7%, 8% and 10% respectively.

Sol. Interest for 1st year $= \text{Rs}\,\frac{5000\times 7\times 1}{100} = \text{Rs } 350$

and, amount at the end of 1st year

$= \text{Rs } 5000 + \text{Rs } 350$

$= \text{Rs } 5350$

= Principal for 2nd year

Interest for 2nd year $= \text{Rs}\ \frac{5350\times 8\times 1}{100}$

$= \text{Rs}\ \frac{42800}{100} = \text{Rs } 428$

and amount at the end of 2nd year

$= \text{Rs } 5350 + \text{Rs } 428$

$= \text{Rs } 5778$

= Principal for 3rd year

Interest for 3rd year $= \text{Rs}\ \frac{5778\times 10\times 1}{100}$

$= \text{Rs}\ \frac{57780}{100} = \text{Rs } 577.80$

and amount due at the end of 3rd year

$= \text{Rs } 5778 + \text{Rs } 577.80$

$= \text{Rs } 6{,}355.80.$

Ex.5 Compute the compound interest on Rs 20,000 for 2 years at 20% per annum when compounded half yearly.

Sol. Here,

Principal (P) = Rs 20000

Rate (r) = 20% per annum

$= \frac{20}{2}$ % or 10% per half year

Time(n) = 2 years

= 4 half years

Since, A $= P\left(1+\frac{r}{100}\right)^n$

$\therefore$ Amount $= \text{Rs } 20000 \times \left(1+\frac{10}{100}\right)^4$

$= \text{Rs } 20000 \times \left(1+\frac{1}{10}\right)^4$

$= \text{Rs } 20000 \times \left(\frac{11}{10}\right)^4$

$= \text{Rs } 20000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10}$

$= \text{Rs } 2 \times 11 \times 11 \times 11 \times 11$

= Rs 29282

$\therefore$ C.I. = A – P

$\therefore$ Compound Interest = Rs (29282 – 20000)

= Rs 9282.

Ex.6 Find the compound interest on Rs 15,625 for 9 months at 16% per annum, compounded quarterly.

Sol. Here, Principal (P) = Rs 15625

Rate (r) = 16% p.a.

= 4% per quarter

Time (n) = 9 months

= 3 quarters

Now, Amount, (A) $= P\left(1+\frac{r}{100}\right)^n$

$= \text{Rs } 15625\left(1+\frac{4}{100}\right)^3$

$= \text{Rs } 15625\left(1+\frac{1}{25}\right)^3$

$= \text{Rs } 15625\left(\frac{26}{25}\right)^3$

$= \text{Rs } 15625 \times \frac{26}{25} \times \frac{26}{25} \times \frac{26}{25}$

$= \text{Rs } 26 \times 26 \times 26$

$= \text{Rs } 17576$

Since, Compound Interest = Amount – Principal

$\therefore$ C.I. = Rs 17576 – Rs 15625

= Rs 1,951.

INVERSE PROBLEMS -

[To find Principal, Time or Rate of Interest]

Ex.7 A certain sum was borrowed at 15% per annum. If at the end of 2 years, Rs 1,290 was compounded as C.I., then find the sum borrowed.

Sol.First Method :

Let the sum be Rs 100

Then, Amount $= \text{Rs } 100\left(1+\frac{15}{100}\right)^2$

$\left[\because \text{Amount} = P\left(1+\frac{r}{100}\right)^n\right]$

$= \text{Rs } 100\left(1+\frac{3}{20}\right)^2$

$= \text{Rs } 100\left(\frac{23}{20}\right)^2$

$= \text{Rs}\left(100\times\frac{23}{20}\times\frac{23}{20}\right) = \text{Rs}\,\frac{529}{4}$

$\therefore$ Compound Interest $= \frac{529}{4} - 100 = \text{Rs}\,\frac{129}{4}$

If C.I. is Rs $\frac{129}{4}$, then the sum borrowed = Rs 100

If C.I. is Rs 1,

then the sum borrowed = Rs $100 \times \frac{4}{129}$

If C.I. is Rs 1290, then the sum borrowed

$= \text{Rs } 100 \times \frac{4}{129} \times 1290$

$= \text{Rs } 100 \times 4 \times 10 = \text{Rs } 4{,}000$

Hence, the sum borrowed is Rs 4,000.

Alternate Method :

Amount $= P\left(1+\frac{r}{100}\right)^n = P\left(1+\frac{15}{100}\right)^2$

$= P\left(1+\frac{3}{20}\right)^2 = P\left(\frac{23}{20}\right)^2$

$= P \times \frac{23}{20} \times \frac{23}{20}$

$\therefore$ C.I. $= \left(P \times \frac{23}{20} \times \frac{23}{20}\right) - P$

$= P\left(\frac{529}{400} - 1\right)$

But, C.I. = Rs 1290

$\therefore \quad P\left(\frac{529}{400} - 1\right) = 1290$

or $P\left(\frac{529-400}{400}\right) = 1290$

or $P\left(\frac{129}{400}\right) = 1290$

or $P = \frac{129 \times 400}{129}$

or $P = 10 \times 400 = 4000$

Hence, Principal = Rs 4,000.

Ex.8 In how many years will Rs 800 amount to Rs 882 at 5% per annum compounded annually ?

Sol. Here, P = Rs 800

A = Rs 882

$r = 5\%$ p.a.

Let number of years be n.

Since, $A = P\left(1+\frac{r}{100}\right)^n$

$\therefore \quad 882 = 800\left(1+\frac{5}{100}\right)^n = 800\left(1+\frac{1}{20}\right)^n$

or $\frac{882}{800} = \left(1+\frac{1}{20}\right)^n$

or $\frac{441}{400} = \left(\frac{21}{20}\right)^n$

or $\left(\frac{21}{20}\right)^n = \left(\frac{21}{20}\right)^n$

$[\because 441 = 21^2$ and $400 = 20^2]$

Since the bases are same on both sides,

hence $n = 2$

Since interest is compounded annually

$\therefore$ Time = 2 years.

Ex.9 Determine the rate percent per annum if
Rs 25,000 amounts to 26,010 in 6 months, interest being compounded quarterly.

Sol. Here, $n = 2$ $[\because$ 6 months = 2 quarters]

Now, $A = P\left(1+\frac{r}{100}\right)^n$, where r is the rate per quarter.

$\therefore \quad 26010 = 25000\left(1+\frac{r}{100}\right)^2$

or $\left(1+\frac{r}{100}\right)^2=\frac{26010}{25000}=\frac{2601}{2500}=\left(\frac{51}{50}\right)^2$

or $\left(1+\frac{r}{100}\right)=\frac{51}{50}$

or $\frac{r}{100}=\frac{51}{50}-1=\frac{51-50}{50}=\frac{1}{50}$

or $r=\frac{1}{50}\times 100=2\%$

Hence, the required rate is 2% p.a.