CUBES AND CUBE ROOTS

CUBES & SOME INTERESTING PATTERN

CUBES

A cube is a solid figure which has all its sides equal. If side of a cube is 1 cm then 27 such cubes can make a big cube of side 3 cm.

So, no. 1, 8, 27, 64, …. are called perfect cube numbers.

Table–1

Number	Cube
1	$1^3 = 1$
2	$2^3 = 8$
3	$3^3 = 27$
4	$4^3 = 64$
5	$5^3 = 125$
6	$6^3 = 216$
7	$7^3 = 343$
8	$8^3 = 512$
9	$9^3 = 729$
10	$10^3 = 1000$

There are only ten perfect cubes from 1 to 1000 and four from 1 to 100 .

Following are the cubes of the numbers from 11 to 20.

Table–2

Number	Cube
11	1331
12	1728
13	2197
14	2744
15	3375
16	4096
17	4913
18	5832
19	6859
20	8000

Results :

1. Cube of even number is also an even number.
2. Cube of an odd number is also an odd number.
3. Unit place of cube of a number whose unit digit is 2, 3, 7, 8 is 8, 7, 3, 2 respectively

SOME INTERESTING PATTERNS

1. Adding consecutive odd numbers :

Observe the following pattern of sums of odd numbers.

$$1 = 1 = 1^3$$
$$3 + 5 = 8 = 2^3$$
$$7 + 9 + 11 = 27 = 3^3$$
$$13 + 15 + 17 + 19 = 64 = 4^3$$
$$21 + 23 + 25 + 27 + 29 = 125 = 5^3$$

Ex.1 How many consecutive odd numbers will be needed to obtain the sum as 10^3 ?

Sol. 10 (91, 93, 95, 97, 99, 101, 103, 105, 107, 109)

2. Prime factors of perfect cube :

Each prime number appears three or multiple of 3 times in its cube.

Eg. $8 = 2 \times 2 \times 2$

Eg. $64 = (2 \times 2) \times (2 \times 2) \times (2 \times 2)$

Eg $125 = (5 \times 5 \times 5) = 5^3 =$ perfect cube number

$\therefore$ a^3 is a perfect cube number.

Ex.2 Is 128 a perfect cube ?

Sol. $128 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times 2$

$= 2^7$

$\because$ power of 2 is not a multiple of 3.

$\therefore$ it is not a perfect cube.

Ex.3 Find the cubes of the following numbers:

(a) 2 (b) 3 (c) 7

(d) 0.9 (e) (–5) (f) – 0.1

Sol. (a) $2 \times 2 \times 2 = 8 \quad \Rightarrow 2^3 = 8$

(b) $3 \times 3 \times 3 = 27 \quad \Rightarrow 3^3 = 27$

(c) $7 \times 7 \times 7 = 343 \quad \Rightarrow 7^3 = 343$

(d) $0.9 \times 0.9 \times 0.9 = 0.729 \quad \Rightarrow (0.9)^3 = 0.729$

(e) $(-5) \times (-5) \times (-5) = -125 \Rightarrow (-5)^3 = -125$

(f) $(-0.1) \times (-0.1) \times (-0.1) = -0.001 \Rightarrow (-0.1)^3 = -0.001$

A natural number is said to be a perfect cube if it is the cube of another natural number.

We know that when odd number of negative factors are multiplied, the product is always negative, so cube can be negative also.

CUBE ROOT

If $2^2 = 4$, then the square root of 4, i.e., $\sqrt{4} = 2$. Similarly, if $2^3 = 8$, then the cube root of 8 is 2. It is written as $\sqrt[3]{8} = 2$. If $3^3 = 27$, then the cube root of 27 is 3. Thus, $\sqrt[3]{27} = 3$.

Note that the symbol $\sqrt{\ }$ implied square root. For our convenience, we omit 2 from $\sqrt[2]{\ }$. But for a cube root, we should use the symbol $\sqrt[3]{\ }$, and it cannot be omitted also we can use $(\ )^{1/3}$ for cube root.