# SQUARES AND SQUARE ROOTS

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If  $p = q^2$  where p and q are integers, then we say that q is the square root of p. For example  $9 = 3^2$ , therefore 3 is the square root of 9, similarly 7 is the square root of 49 and 12 is the square root of 144. We can say that if p is a perfect square then its square root is an integer and if p is not a perfect square then it does not have an integral square root.

Symbolically, square roots of a positive number 'n' is written as  $\sqrt{n}$  or  $\sqrt[2]{n}$  or  $(n)^{\frac{1}{2}}$ . Therefore,

 $\sqrt{16} = 4$  or  $\sqrt[2]{16} = 4$  or  $(16)^{\frac{1}{2}} = 4$ .

#### **Properties of Square Roots**

Based upon the properties of square number discussed, we have the following properties of square roots

#### **Property 1**

If the units digit of a number is 2, 3, 7 or 8, then it does not have a square root in N (the set of natural numbers).

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Explanation: By property 1, a number having 2, 3, 7 or 8 at unit's place cannot be a perfect square. Hence, a number having 2, 3, 7 or 8 at units place does not have a square root in N.
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#### **Property 2**

If a number ends in an odd number of zeros, then it does not have a square root. If a square number is followed by an even number of zeros, it has a square root in which the number of zeros in the end is half the number of zeros in the number.

**Explanation:** By property 2, the number of zeros at the end of a perfect square is always even and is twice the number of zeros at the end of the number.

### Property 3

The square root of an even square number is even and that square root of an odd square number is odd.

**Explanation:** By property 3, the squares of even numbers are even numbers and that of odd numbers are odd numbers.

### **Property 4**

If a number has a square root in N, then its units digit must be 0, 1, 4, 5, 6 or 9.

**Explanation :** By property 6, the units digits of the square and square root are related as

below:

Units digit of square	0	1	4	5	6	9
Units digit of square root	0	1 or 9	2 or 8	5	4 or 6	3 or 7

## Property 5

Negative numbers have no square root in the system of rational numbers.

### Explanation:

We have,  $2^2 = 4$ ,  $3^2 = 9$ ,  $4^2 = 16$  and so on. Also,  $(-2)^2 = (-2) \times (-2) = 4$ ,

 $(-3)^2 = (-3) \times (-3) = 9, (-4)^2 = (-4) \times (-4) = 16$  and so on. This means that the square of a number whether positive or negative is always positive. Consequently, negative numbers are not perfect squares. Hence, negative numbers have no square roots.

# Property 6

The sum of first n odd natural numbers is  $n^2$  i.e.

 $1 + 3 + 5 + 7 + \dots + (2n - 1) = n^2$ 

