SQUARES AND SQUARE ROOTS

SOME MORE INTERESTING PATTERNS

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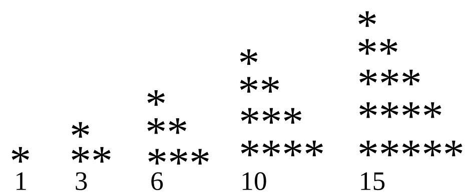
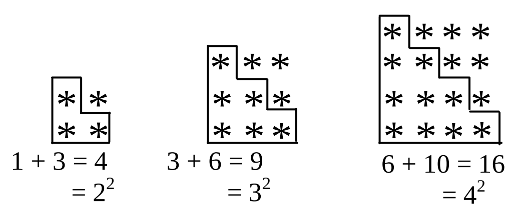
1. Adding triangular numbers.

Triangular Numbers :

Numbers whose dot patterns can be arranged as triangles.



If we combine two consecutive triangular numbers, we get a square number, like



2. Numbers between square numbers

We can find some interesting pattern between two consecutive square numbers.

1 $(=1^2)$

1, 2, 3, 4 $(= 2^2)$

1 and 4 are perfect square

4, 5, 6, 7, 8, 9 $(= 3^2)$

4 and 9 are perfect square

9, 10, 11, 12, 13, 14, 15, 16 $(= 4^2)$

9 and 16 are perfect square

16, 17, 18, 19, 20, 21, 22, 23, 24, 25 $(= 5^2)$

16 and 25 are perfect square

Between 1^2 (=1) and 2^2 (= 4) there are two (i.e., 2×1)

non square numbers 2, 3.

Between 2^2 (= 4) and 3^2 (= 9) there are four (i.e., 2×2) non square numbers 5,6,7,8.

Now, $3^2 = 9$, $4^2 = 16$

Therefore. $4^2 - 3^2 = 16 - 9 = 7$

Between 9 $(= 3^2)$ and 16 $(= 4^2)$ the numbers are 10, 11, 12, 13, 14, 15 that is, six non square numbers which is 1 less than the difference of two squares.

We have $4^2 = 16$ and $5^2 = 25$

Therefore, $5^2 - 4^2 = 9$

Between 16 $(= 4^2)$ and 25 $(=5^2)$ the numbers are 17, 18, ..., 24 that is, eight non square numbers which is 1 less than the difference of two squares.

In general we can say that there are 2n non perfect square numbers between the squares of the numbers n and (n + 1).

3. Adding odd numbers

1 [one odd number] $= 1 = 1^2$

1 + 3 [sum of first two odd numbers] $= 4 = 2^2$

1 + 3 + 5 [sum of first three odd numbers] $= 9 = 3^2$

1 + 3 + 5 + 7 [...] $= 16 = 4^2$

$1 + 3 + 5 + 7 + 9$ [.....] $= 25 = 5^2$

$1 + 3 + 5 + 7 + 9 + 11$ [....] $= 36 = 6^2$

So we can say that the sum of first n odd natural numbers is n^2.

Looking at it in a different way, we can say : 'If the number is a square number, it has to be the sum of successive odd numbers starting from 1.

Consider the number 25. Successively subtract 1,3,5,7,9,..... from it

(i) 25 – 1 = 24 (ii) 24 – 3 = 21 (iii) 21 – 5 = 16

(iv) 16 – 7 = 9 (v) 9 – 9 = 0

This means, 25 = 1 + 3 + 5 + 7 + 9. Also, 25 is a perfect square.

Now consider another number 38, and again do as above.

(i) 38 – 1 = 37 (ii) 37 – 3 = 34

(iii) 34 – 5 = 29 (iv) 29 – 7 = 22

(v) 22 – 9 = 13 (vi) 13 – 11 = 2

(vii) 2 – 13 = – 11

This shows that we are not able to express 38 as the sum of consecutive odd numbers starting with 1. Also, 38 is not a perfect square.

So we can also say that if a natural number cannot be expressed as a sum of successive odd natural numbers starting with 1, then it is not a perfect square.

We can use this result to find whether a number is a perfect square or not.

4. A sum of consecutive natural numbers

Consider the following

$3^2 = 9 = 4 + 5$

Here, 4 = First number = $\frac{3^2 - 1}{2}$;

5 = Second number = $\frac{3^2 + 1}{2}$

$5^2 = 25 = 12 + 13$

$7^2 = 49 = 24 + 25$

$9^2 = 81 = 40 + 41$

$11^2 = 121 = 60 + 61$

$15^2 = 225 = 112 + 113$

Note : We can express the square of any odd number as the sum of two consecutive positive integers.

5. Product of two consecutive even or odd natural numbers

$11 \times 13 = 143 = 12^2 - 1$

Also $11 \times 13 = (12 - 1) \times (12 + 1)$

Therefore, $11 \times 13 = (12 - 1) \times (12 + 1) = 12^2 - 1$

Similarly, $13 \times 15 = (14 - 1) \times (14 + 1) = 14^2 - 1$

$29 \times 31 = (30 - 1) \times (30 + 1) = 30^2 - 1$

$44 \times 46 = (45 - 1) \times (45 + 1) = 45^2 - 1$

So in general we can say that $(a + 1) \times (a - 1) = a^2 - 1$

6. Some more patterns in square numbers

Observe the squares of numbers ; 1, 11, 111 …… etc. They give a beautiful pattern :

1^2	1
11^2	121
111^2	12321
1111^2	1234321
11111^2	123454321
11111111^2	123456787654321

Another interesting pattern.

$7^2 = 49$

$67^2 = 4489$

$667^2 = 444889$

$6667^2 = 44448889$

$66667^2 = 4444488889$

$666667^2 = 444444888889$