SQUARES AND SQUARE ROOTS

FINDING THE SQUARE OF A NUMBER

TO FIND THE SQUARE OF A NUMBER BY THE COLUMN METHOD

The procedure given below explains the application of the column method to find the square of a two digit number.

Procedure

If the given two digit number is of the form ab, where 'a' is the digit in ten place and 'b' is the digit in the units place, then calculate.

a^2	$2ab$	b^2
Col I	Col II	Col III

Now, we have three columns. Consider b^2 in Column III. Underline the units digit of b^2.

Add the tens digit of b^2, if any, to 2ab in Column II and then underline the units digit in Column II.

After underlining the units digit in Column II, add the non-underlined part of column II, if any, to a^2 in Column I.

Underline the number thus obtained in Column I.

The underlined digits when written in the same order as a single number gives the required square.

Ex.1 To find the square of 64.

Sol. Here a = 6 and b = 4

	Column I	Column II	Column III
	A^2	2ab	b^2
Step(1)	36	$4\underline{8}$	$1\underline{6}$
Step(2)	36	$4\underline{2}$	$\underline{6}$

Step (3) 40 9 6

Units digit in Column III is 6.

On adding the tens digit in column III to the number in Column II, we get $48 + 1 = 49$

Now the units digit in Column II is 9.

On adding the tens digit in Column II, to the number in the Column I,

we get $36 + 4 = 40$

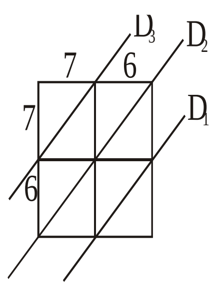
$\therefore$ The square of 64 in 4096.

Note : If the number of digits in the number to be squared is more than two, the use of column method should be avoided, as the method then becomes very difficult of apply.

TO FIND THE SQUARE OF A NUMBER BY THE DIAGONAL METHOD

(i) Initially, we draw a square. If the number of digits in the given number is 2, then we divide the square into 4 sub-squares and in case the number of digits in the given number is 3, we divide the square into 9 sub-squares and so on.

(ii) Say, the given number is 76 (a two digit number). Construct the diagonals and write the digits of the given number as shown in the figure given below.



(iii) Now multiply each digit on the left of the square with each digit on the top of the column one by one. Write the product in the corresponding sub-square.

(iv) If the number obtained is a single digit number, then write it below the diagonal.

(v) If the number obtained is a two digit number, then write the tens digit above the diagonal and the units digit below the diagonal.

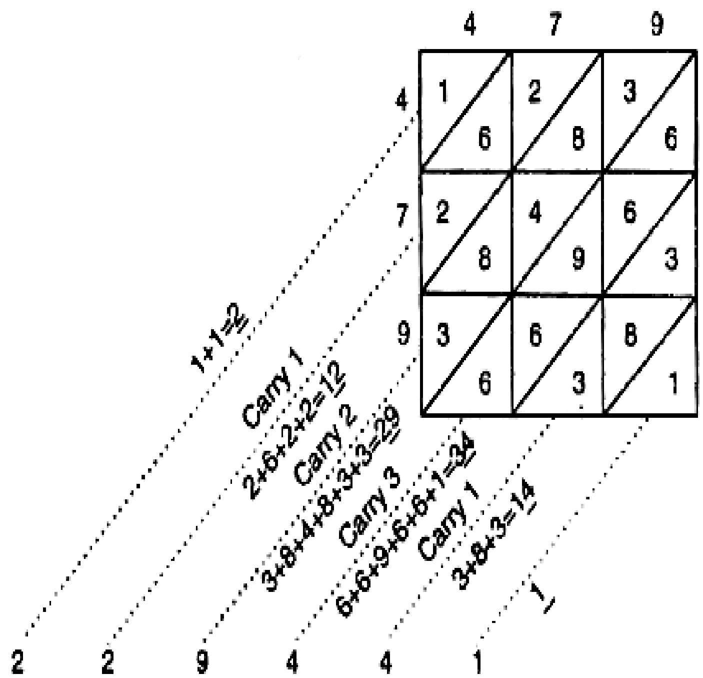
(vi) The numbers in empty places are taken as zero.

(vii) Starting below the lowest diagonal add the digits along the diagonals so obtained. Underline the units digit of the sum and carry over the tens digit, if any, to the diagonal above.

(viii) The underlined unit digits together with, all the digits in the sum obtained above the top most diagonal, give the square of the number.

Ex.2 To find the suqare of 479 by digonal method.

Sol



Thus, square of 479 is 229441.

Ex.3 To find the square of 58.

Sol In the figure, the number below that lowest diagonal is 4.

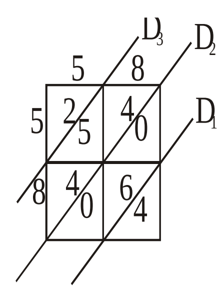
Sum of the numbers in between D_1 and D_2 is $0 + 6 + 0 = 6$.

Sum of the numbers in between D_2 and D_3 is $4 + 5 + 4 = 13$.

The units digit of the sum obtained between D_2 and D_3 is 3.

Add the tens digit number of the number 13 to numbers above D_3.

So the sum above D_3 is $2 + 1 = 3$.



$\therefore$ Required square is the combination of all the unit digits in all diagonals = 3364.

FINDING SQUARES OF THE NUMBERS THAT FOLLOW A FIXED PATTERN

Observe the following pattern.

$11^2 = 121$

$1\underline{0}1^2 = 1\ \underline{0}\ 2\ \underline{0}\ 1$

$1\underline{00}1^2 = 1\ \underline{0\ 0}\ 2\ \underline{0\ 0}\ 1$

Ex.4 Find the value of 10001^2.

Sol From the above patter, we have $10001^2 = 100020001$

observe the following patter.

$9^2 = 81$

$\underline{9}9^2 = \underline{9}801$

$\underline{99}9^2 = \underline{99}8001$

Ex.5 Find the value of 9999^2.

Sol From the above pattern $9999^2 = 99980001$

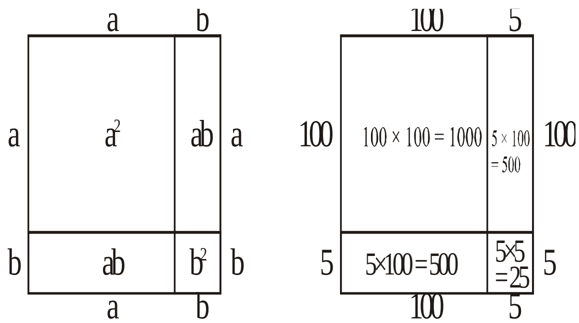
TO FIND THE SQUARE OF NUMBER BY USING $(a + b)^2$ OR $(a - b)^2$

Visual Method

In the column method we have used the algebraic identity $(a + b)^2 = a^2 + 2ab + b^2$ to compute the square of a two digit number. The square of a positive integer can also be computed by closely following the visual representation of $(a + b)^2$. In order to represent $(a + b)^2$, we draw a square of side $a + b$ and divide it into two rectangles of size $a \times (a + b)$ and $b \times (a + b)$ by drawing a vertical line as shown in Fig. We also draw a horizontal line divide the square into two rectangles of size $(a + b) \times b$ and $(a + b) \times a$ as shown in Fig

(A). These two lines divide the square into four parts, namely, two squares of size a $\times$ a and b $\times$ b and two rectangles of size a $\times$ b and b $\times$ a. The sum of the areas of these four parts is

$a \times a + a \times b + b \times a + b \times b = a^2 + 2ab + b^2 = (a + b)^2$



(A) (B)

We use this visual representation of $(a + b)^2$ to find the square of a number.

Suppose we wish to find the square of 105.

We have, $105 = 100 + 5$

So, we draw a square of side 105 units and divide it into four parts as shown in Fig.(B).

The sum of the areas of these four parts is the square of 105.

$\therefore\ 105^2 = 10000 + 500 + 500 + 25 = 11025$

Note : this method is limited to very few numbers.

Ex.6 Find $(102)^2$.

Sol $(102)^2 = (100 + 2)^2 = (100)^2 + 2(100)\ (2) + (2)^2 = 10000 + 400 + 4 = 10404$