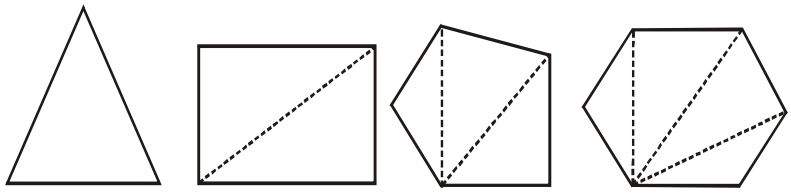
UNDERSTANDING QUADRILATERALS

ANGLE SUM PROPERTY

ANGLE AND PROPERTY

Observe the following figures, Each figure is divided into triangles.



A triangle can be divided into one triangle, a quadrilateral into 2 triangles, a pentagon into

3 triangles, a hexagon into 4 triangles.

Thus, a polygon of n sides can be divides into (n – 2) triangles.

Sum of the angles of a triangle $= 180° = (3 - 2) \times 180°$

Sum of the angles of a quadrilateral $= 360° = (4 - 2) \times 180°$

Sum of the angles of a pentagon $= 540° = (5 - 2) \times 180°$

Sum of the angles of a hexagon $= 720° = (6 - 2) \times 180°$

So, sum of the angles of a polygon of n-sides $= (n - 2) \times 180° = (n - 2) \times 2 \times 90°$

$= (2n - 4) \times 90° = (2n - 4)$ right angles

Thus, Sum of the angles of a polygon of n-sides $= (2n - 4)$ right angles

$= (n - 2) \times 180°$

Ex.1 If two angles of a triangle are 40° & 58° then find the third angle.

Sol. $\because$ The sum of all angles $= 180°$ (A.S.P.)

$40° + 58° +$ Third angle $= 180°$

$\therefore$ Third angle $= 180° - 98° = 102°$

Ex.2 If two angles of a hexagon are right angles & rest angles are same to each other then find the value of one of the other angles.

Sol. Let the other each angle = x°

$\therefore 90° + 90° + x + x + x + x = (n - 2)\ 180°$

$\Rightarrow 180 + 4x = (6 - 2)180°$

$\Rightarrow 4x = 720 - 180°$

$\Rightarrow \quad x = \frac{540}{4} = 135°.$

Ex.3 Find the maximum exterior and minimum interior angle of regular polygon.

Sol. $\because$ Minimum number of sides in a regular polygon is 3 (equilateral Δ)

$\therefore$ each angle = x° (Let)

$\therefore 3x = 180 \Rightarrow x = 60°$

$\therefore$ minimum value of interior angle = 60°

$\therefore$ maximum exterior angle = 120°

(by linear pair).

Ex.4 The angles of a quadrilateral are in ratio 1 : 3 : 7 : 9 find the measure of each angle.

Sol. Let angles are x°, 3x°, 7x°, 9x°

$\therefore x + 3x + 7x + 9x = 360°$ (A.S.P.)

$\Rightarrow 20x = 360$

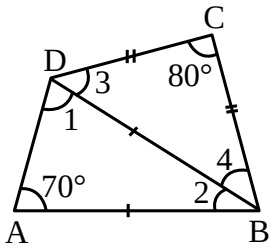
$x = 18$

$\therefore$ angles are $18, 18 \times 3, 18 \times 7, 18 \times 9$

$= 18°, 54°, 126°, 162°$

Verification $18° + 54° + 126° + 162° = 360°$

Ex.5 Find the angles of quadrilateral ABCD, in given figure.



Sol. $\because$ AB = BD (in ΔADB)

$\therefore \angle 1 = 70°$

$\therefore \angle 1 + 70° + \angle 2 = 180°$ (A.S.P.)

$\Rightarrow 70 + 70 + \angle 2 = 180°$

$\Rightarrow \angle 2 = 180° - 140° = 40°$

Also in ΔDCB

DC = CB

$\therefore \angle 3 = \angle 4 = y$

$\therefore y + y + 80 = 180°$ (A.S.P.)

$\Rightarrow 2y = 180 - 80$

$\Rightarrow y = \frac{100}{2} = 50°$

$\therefore \angle 3 = \angle 4 = 50°$

$\therefore \angle ABC = \angle 2 + \angle 4 = 40° + 50° = 90°$

& $\angle ADC = \angle 1 + \angle 3 = 70° + 50° = 120°$

$\therefore$ angles are 70°, 90°, 80°, 120°.