LINEAR EQUATION IN ONE VARIABLE

APPLICATION OF LINEAR EQUATION TO PRACTICAL PROBLEM

Applications of Linear Equations to Practical Problems

The following steps should be followed to solve a word problem:

Step-I Read the problem carefully and note what is given and what is required.

Step-II Denote the unknown quantity by some letters, say x, y, z, etc.

Step-III Translate the statements of the problem into mathematical statements.

Step-IV Using the condition(s) given in the problem, form the equation.

Step-V Solve the equation for the unknown.

Step-VI Check whether the solution satisfies the equation.

Ex.1 A number is such that it is as much greater than 84 as it is less than 108. Find it.

Sol. Let the number be x. Then, the number is greater than 84 by x – 84 and it is less than 108 by 108 – x. [Given]

$\therefore \quad x - 84 = 108 - x$

$\Rightarrow \quad x + x = 108 + 84$

$\Rightarrow \quad 2x = 192 \Rightarrow \frac{2x}{2} = \frac{192}{2} \Rightarrow x = 92$

Hence, the number is 96.

Ex.2 A number is 56 greater than the average of its third, quarter and one-twelfth. Find it.

Sol. Let the number be x. Then,

One third of x is $= \frac{1}{3}$ x, Quarter of x is $= \frac{x}{4}$,

One-twelfth of x is $= \frac{x}{12}$

Average of third, quarter and one-twelfth of

x is $= \frac{\left(\frac{x}{3}+\frac{x}{4}+\frac{x}{12}\right)}{3} = \frac{1}{3}\left(\frac{x}{2}+\frac{x}{4}+\frac{x}{12}\right)$

It is given that the number x is 56 greater than the average of the third, quarter and one-twelfth of x.

$\therefore \quad x = \frac{1}{3}\left(\frac{x}{3}+\frac{x}{4}+\frac{x}{12}\right)+56$

$\Rightarrow \quad x = \frac{x}{9}+\frac{x}{12}+\frac{x}{36}+56$

$\Rightarrow \quad x - \frac{x}{9} - \frac{x}{12} - \frac{x}{36} + 56$

$\Rightarrow \quad 36x - 4x - 3x - x = 36 \times 56$

[Multiplying both sides by 36 i.e., the L.C.M. of 9, 12 and 36]

$\Rightarrow \quad 36x - 8x = 36 \times 56$

$\Rightarrow \quad 28x = 36 \times 56$

$\Rightarrow \quad \frac{28x}{28} = \frac{36 \times 56}{28}$

[Dividing both sides by 28]

$\Rightarrow \quad x = 36 \times 2$

$\Rightarrow \quad x = 72$

Hence, the number is 72.

Ex.3 A number consists of two digits whose sum is 8. If 18 is added to the number, the digits are interchanged. Find the number

Sol. Let one's digit be x.

Since the sum of the digits is 8. Therefore, ten's digit = 8 – x.

$\therefore$ Number = $10 \times (8 - x) + x = 80 - 10x + x$

$= 80 - 9x$ ... (i)

Now,

Number obtained by reversing the digit

$= 10 \times x + (8 - x) = 10x + x - x = 9x + 8.$

It is given that if 18 is added to the number its digits are reversed.

$\therefore$ Number + 18 = Number obtained by reversing the digits

$\Rightarrow$ $80 - 9x + 18 = 9x + 8$

$\Rightarrow$ $98 - 9x = 9x + 8 \Rightarrow 98 - 8 = 9x + 9x$

$\Rightarrow$ $90 = 18x \quad \Rightarrow \quad \frac{18x}{18} = \frac{90}{18}$

$\Rightarrow$ $x = 5$

Putting the value of x in (i), we get

Number = $80 - 9 \times 5 = 80 - 45 = 35$

Ex.4 Divide 34 into two parts in such a way that $\left(\frac{4}{7}\right)^{th}$ of one part is equal to $\left(\frac{2}{5}\right)^{th}$ of the other.

Sol. Let one part be x. Then, other part is (34 – x). It is given that

$\left(\frac{4}{7}\right)^{th}$ of one part = $\left(\frac{2}{5}\right)^{th}$ of the other part

$\Rightarrow \quad \frac{4}{7}x = \frac{2}{5}(34 - x) \Rightarrow 20x = 14(34 - x)$

[Multiplying both sides by 35, the LCM of 7 and 5]

$\Rightarrow \quad 20x = 14 \times 34 - 14x$

$\Rightarrow \quad 20x + 14x = 14 - 34$

$\Rightarrow \quad 34x = 14 \times 34$

$\Rightarrow \quad \frac{34x}{34} = \frac{14 \times 34}{34}$

[Dividing both sides by 34]

$\Rightarrow \quad x = 14$

Hence, the two parts are 14 and 34 – 14 = 20

Ex.5 The numerator of a fraction is 4 less that the denominator. If 1 is added to both its numerator and denominator, it becomes 1/2. Find the fraction.

Sol. Let the denominator of the fraction be x. Then,

Numerator of the fraction = x – 4

$\therefore \quad$ Fraction $= \frac{x-4}{x}$ ...(i)

If 1 is added to both its numerator and denominator, the fraction becomes $\frac{1}{2}$

$\therefore \quad \frac{x-4+1}{x+1} = \frac{1}{2}$

$\Rightarrow \quad \frac{x-3}{x+1} = \frac{1}{2}$

$\Rightarrow \quad 2(x - 3) = x + 1$

[Using cross-multiplication]

$\Rightarrow \quad 2x - 6 = x + 1$

$\Rightarrow \quad 2x - x = 6 + 1$

$\Rightarrow \quad x = 7$

Putting $x = 7$ in (i), we get

Fraction $= \frac{7-4}{7} = \frac{3}{7}$

Hence, the given fraction is $\frac{3}{7}$.