PLAYING WITH NUMBERS

REVERSING THE DIGITS AND FIND THE DIGIT

GAMES WITH NUMBERS

Reversing the Digits of a 2-digit Number:

Consider a 2-digit number ab whose expanded form is $10a + b$. On reversing its digits, we get another number $ba = 10b + a$.

On adding the two numbers, we get

$$ab + ba = 10a + b + 10b + a$$

$$= 11a + 11b = 11(a + b)$$

$$\Rightarrow \quad \frac{ab+ba}{11} = a + b \quad \text{or} \quad \frac{ab+ba}{(a+b)} = 11$$

Thus, the sum of any 2 digit number ab and the number ba obtained by reversing its digits, is completely divisible by

(i) 11 and the quotient is $a + b$

(ii) $a + b$ and the quotient is 11.

Ex.1 Without performing the actual division and addition, find the quotient when the sum of 87 and 78 is divided by (i) 11 (ii) 15.

Sol. 87 and 78 are the two numbers such that one can be obtained by interchanging the digits of the other.

(i) If we divide their sum by 11, the quotient is the sum of the digits i.e.,

$$\frac{87+78}{11} = 7+8 = 15$$

(ii) If we divide their sum by the sum of the digits, we get 11 as the quotient.

Or $\frac{87+78}{7+8} = 11$ or $\frac{87+78}{15} = 11$

Again consider

$ab = 10a + b$ and $ba = 10b + a$

(i) If $a > b$, then the difference between the numbers is

$ab - ba = 10a + b - 10b - a$

$= 9a - 9b = 9(a - b)$

$\Rightarrow \frac{ab-ba}{9} = a - b$ or $\frac{ab-ba}{a-b} = 9$

(ii) If $b > a$, then difference between the numbers is

$ba - ab = 10b + a - 10a - b$

$= 9b - 9a = 9(b - a)$

$\Rightarrow \frac{ba-ab}{9} = b - a$ or $\frac{ba-ab}{b-a} = 9$

(iii) If $a = b$, then $ab - ba = 0$

$\Rightarrow \frac{ab-ba}{9} = 0$

Thus, the difference of any 2-digit number ab and the number obtained ba by reversing its digits, is completely divisible by

(i) 9 and the quotient is the difference of the digits

(ii) the difference of the digits, $(a \neq b)$ and the quotient is 9.

Ex.2 Without performing actual subtraction and division, find the quotient which the difference of 94 and 49 is divided by : (i) 9 (ii) 5

Sol. 94 and 49 are the two numbers, such that one can be obtained by interchanging the digits of the other.

Thus,

(i) If we divided their difference by 9, the quotient is the difference of the digits

Or $\frac{94-49}{9} = 9 - 4 = 5$

(ii) If we divide their difference by the difference of the digits, we get 9 as the quotient.

Or $\frac{94-49}{9-4} = 9$ or $\frac{94-49}{5} = 9$

REVERSING THE DIGITS OF A 3-DIGIT NUMBER

Consider a 3-digit number abc whose expanded form is 100a + 10b + c. On reversing its digit, we get another number,

cba = 100c + 10b + a

(i) If a > c, then the difference of the number is

$abc - cba = 100a + 10b + c - 100c - 10b - a$

$= 99a - 99c = 99(a - c)$

$\Rightarrow \frac{abc-cba}{99}$ a -c or $\frac{abc-cba}{a-c} = 99$

(ii) If c > a, then the difference of the numbers is

$cba - abc = 100c + 10b + a - 100a - 10b - c$

$= 99c - 99a = 99(c - a)$

$\Rightarrow \frac{cba-abc}{99} = c - a$ or $\frac{cba-abc}{c-a} = 99$

(iii) If a = c, then the difference of the numbers is

$abc - cba = 100a + 10b + c - 100c - 10b - a = 0$

$\Rightarrow \frac{abc-cba}{99} = 0$

Thus, the difference of any 3-digit number abc and the number cba by reversing its digits, is exactly divisible by

(i) 99 and the quotient is the difference of hundreds and ones digit of the number.

(ii) a – c or c – a, (a ¹ c) and the quotient is 99.

Ex.3 Without performing actual subtraction and division, find the quotient when the difference of 589 and 985 is divided by : (i) 99 (ii) 4

Sol. (i) 589 and 985 are the two 3-digit numbers such that one can be obtained by reversing the digits of the correct.

Thus, (i) If we divide their difference by 99, the quotient is the difference of hundreds and ones digits.

or $\frac{985-589}{99} = 9 - 5 = 4$

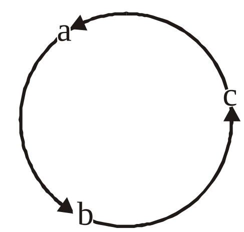
(ii) If we divide their difference by the difference of hundreds difference by the difference of hundreds and ones digits, the quotient is 99.

or $\frac{985-589}{9-5} = 99$ or $\frac{985-589}{4} = 99$

FORMING 3-DIGIT NUMBERS WITH GIVEN THREE DIGITS

Consider a 3 - digit number abc = 100a + 10b + c.

By changing the order of its digits in cyclic order, we get two more 3-digit number.



bca = 100b + 10c + a

and cab = 100c + 10a + b

On adding these three numbers, we get

abc + bca + cab = 100a + 10b + c + 100b + 10c + a + 100c + 10a + b

= 111a + 111b + 111c

$\Rightarrow$ abc + bca + cab = 111 (a + b + c) = 3 × 37 (a + b + c)

Sol. (i) $\frac{abc+bca+cab}{111} = a+b+c$

(ii) $\frac{abc+bca+cab}{a+b+c} = 111$

(iii) $\frac{abc+bca+cab}{37} = 3(a+b+c)$

(iv) $\frac{abc+bca+cab}{3} = 37(a+b+c)$

(v) $\frac{abc+bca+cab}{3(a+b+c)} = 37$

(vi) $\frac{abc+bca+cab}{37(a+b+c)} = 3$

Ex.4 Without performing actual addition and division, find the quotient when the sum of 584, 845 and 458 is divisible by :

(i) 17 (ii) 37 (iii) 111

(iv) 3 (v) 51 (vi) 629

Sol. 584, 845, 458 are three numbers obtained when the digits 5, 8 and 4 are arranged in the cyclic order.

So, the quotient when the sum of these numbers is divided by :

(i) $5 + 8 + 4 = 17$ is 111

(ii) 37 is $3 \times (5 + 8 + 4)$ i.e., 51

(iii) 111 is $(5 + 8 + 4)$ i.e., 17

(iv) 3 is $37 \times (5 + 8 + 4)$ i.e., 629

(v) $3 \times (5 + 8 + 4) = 51$ is 37

(vi) $37 \times (5 + 8 + 4) = 629$ is 3.