PLAYING WITH NUMBERS

DIVISIBILITY TEST

TEST OF DIVISIBILITY

- ### TEST OF DIVISIBILITY BY 2.

A number is divisible by 2 if its units digit is even, i.e., if its units digit is any of the digits 0, 2, 4, 6 or 8.

For a number in the generalized form:

(i) **A two-digit number 10a + b :** is divisible by 2 if 'b' is any of the digits 0, 2,4,6 or 8.

(ii) **A three-digit number 100a + 10b + c :** is divisible by 2 if 'c' is any of the digits 0, 2, 4, 6 or 8.

Ex. The numbers 12, 68, 120, 854 are all divisible by 2.

- ### TEST OF DIVISIBILITY BY 3.

A number is divisible by 3, if the sum of its digits is divisible by 3. For a number in the generalized form:

(i) **A two-digit number 10a + b :** is divisible by 3 if (a + b) is divisible by 3.

(ii) **A three-digit number 100a + 10b + c :** is divisible if (a + b + c) is divisible by 3.

Ex. 21, 54, 123, 351 are all divisible by 3 but none of the numbers 22, 56, 76, 359, 835 divisible by 3.

- ### TEST OF DIVISIBILITY BY 5.

A number is divisible by 5, if its units digit is either 0 or 5. For a number in the generalized form :

(i) **A two-digit number l0a + b :** is divisible by 5 if 'b' is either 0 or 5.

(ii) **A three-digit number 100a + 10b + c :** is divisible by 5, if 'c' is divisible by 5.

Ex. The numbers 15, 80, 110 are all divisible by 5.

• TEST OF DIVISIBILITY BY 9.

A number is divisible by 9 if the sum of its digits is divisible by 9. For a number in the generalized form:

(i) **A two-digit number $10a + b$:** is divisible by 9 if '$a + b$' is divisible by 9.

(ii) **A three-digits number $100a + 10b + c$:** is divisible by 9 if '$a + b + c$' is divisible by 9.

Ex. The numbers 18, 27, 225, 801 are all divisible by 9.

• TEST OF DIVISIBILITY BY 10.

A number is divisible by 10, if its units digit is 0. For a number in the generalized form :

(i) **A two-digit number $10a + b$:** is divisible by 10, if 'b' is equal to 0.

(ii) **A three-digit number $100a + 10b + c$:** is divisible by 10, if 'c' is equal to 0.

Ex. The numbers 20, 70, 580, 900 are divisible by 10.

• TEST OF DIVISIBILITY BY 11.

A number is divisible by 11, if the difference of its digits in odd places and the sum of its digits in even places is either 0 or a multiple of 11.

		(i)	(ii)	(iii)
Ex.	3-digit → 264 →	2	6	4

Sum of the digits in odd places – Sum of the digits in even places.

$(2 + 4) - 6 = 0$

264 is divisible by 11

Ex.	61809 →	6	1	8	0	9
		(i)	(ii)	(iii)	(iv)	(v)

$(6 + 8 + 9) - (1 + 0) \rightarrow 23 - 1 = 22$

61809 is divisible by 11.

DIVISOR	DIVISIBLITY CONDITION	EXAMPLES
4	The last two digits divisible by 4.	40832 32 is divisible by 4
6	It is divisible by 2 and by 3.	1458 $1 + 4 + 5 + 8 = 18$ So it is divisible by 3 and the last digit is even, hence the number is divisible 6
7	Add 5 times the last digit to the rest.	483 $48 + (3 \times 5) = 63$
8	The number formed by last 3 digits of given number should be visible by 8	34152 Examine divisibility of just 152
12	It is divisible by 1 and by 4.	$324 : (32 \times 2) - 4 = 60$
13	Add the digits in alternate blocks of three from right to left, then subtract the two sms.	$2, 911, 272 : (2 + 272) + 911 = 637$
15	It is divisible by 3 and by 5.	390 : It is divisible by 3 and by 5.
19	Add twice the last digit to the rest	$437 : 43 + (7 \times 2) = 57$

Ex.1 Find the least value of x for which 7x5462 is divisible by 9 :

Sol. Let the required value be x then $(7 + x + 5 + 4 + 6 + 2) = (24 + x)$ is divisible by 9.

Ex.2 Find the least value of x for which 4832x18 is divisible by 11.

Sol. (Sum of digits at odd places) – (Sum of digits at even places)

$(8 + x + 3 + 4) - (1 + 2 + 8) = (4 + x)$

which should be divisible by 11, $x = 7$

Ex.3 What is the unit digit in the product $(684 \times 759 \times 413 \times 676)$?

Sol. Unit digit in the given product = unit digit in the product $(4 \times 9 \times 3 \times 6) = 8$.

Ex.4 What is the unit digit in the product $(3547)^{153} \times (251)^{72}$?

Sol. Required digit = unit digit in $(7^{153} \times 1^{72})$

Now, 7^4 gives unit digit 1 and $1^{72} = 1$

$(7^{153} \times 1^{72}) = [(7^4)^{38} \times 7 \times 1]$

Required unit digit = $(1 \times 7 \times 1) = 7$.

Ex.5 What is the unit digit in $(264)^{102} + (264)^{103}$?

Sol. $(264)^{102} + (264)^{103} = (264)^{102} \times (1 + 264) = (264)^{102} \times 265$

Required unit digit = unit digit in $[4^{102} \times 5] = [(4^4)^{25} \times 4^2] \times 5 = 6 \times 6 \times 5 = 0$

Ex.6 Find the total number of prime factors in the product $(4^{11} \times 7^5 \times 11^2]$?

Sol. $(2 \times 2)^{11} \times 7^5 \times (11)^2$

$\Rightarrow (22)^{11} \times 7^5 \times (11)^2$

$\Rightarrow 2^{22} \times 7^5 \times 11^2$

Required number of factors = $(22 + 5 + 2) = 29$

Ex.7 Find the remainder when 2^{31} is divided by 5 ?

Sol. $2^{31} = (2^{10} \times 2^{10} \times 2^{10}) \times 2$

$\Rightarrow (2^{10})^3 \times 2 = (1024)^3 \times 2$

Unit digit in 2^{31} = unit digit in $[(1024)^3 \times 2] = 4 \times 2 = 8$

Now, 8 when divided by 5 gives 3 as remainder

2^{31} when divided by 5 given remainder = 3.