FACTORISATION

FACTORISATION USING IDENTITIES

Factorisation with the help of Algebraic Identities

Let us recall the following algebraic identities:

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$; $(x + y)(x - y) = x^2 - y^2$

$(x + a)(x + b) = x^2 + (a + b)x + ab$

Thus, we can say that

Factors of $x^2 + 2xy + y^2$ are $x + y$ and $x + y$

Factor of $x^2 - 2xy + y^2$ are $x - y$ and $x - y$

Factors of $x^2 + (a + b)x + ab$ are $x + a$ and $x + b$

On the basis of the above discussion let us deal with the following examples of factorisation.

A. Factorisation by using the Identities $x^2 \pm 2xy + y^2 = (x \pm y)^2$

Ex.1 Factorise : $25x^2 - 20x + 4$

Sol: $25x^2 - 20x + 4$

$\downarrow \quad \uparrow \quad \downarrow$

$(5x)^2 - 2 \times 5x \times 2 + (2)^2$

$(5x - 2)^2 = (5x - 2)(5x - 2)$

Note: that in these two examples in second step two arrows are downward and one arrow is upward. This shows that in the second step first we write 1^{st} and 3^{rd} terms on the basis of given terms and then write the middle term to complete the formula and then compare it with given middle term.

B. Factorisation by Using the Identity $x^2 - y^2 = (x + y)(x - y)$

Ex.2 Factorize $121x^2 - 81y^2$

Sol. $121x^2 - 81y^2 = (11x)^2 - (9y)^2$

(Using identity $x^2 - y^2 = (x - y)(x + y)$

$= (11x - 9y)(11x + 9y)$

C. Factorisation of Trinomial $x^2 + mx + n$

By splitting up the middle terms or factorisation by using the identity:

$x^2 + (a + b)x + ab = (x + a)(x + b)$.

We can find out two numbers a and b positive or negative, such that (a + b) is the same as the coefficient of x whereas the product ab is equal to the constant term in the given expression.

Let us consider examples to explain the above process.

Ex.3 Factorise $x^2 + 6x + 8$.

Sol. Here we have to find out two numbers a and b such that :

$a + b = 6$ (the coefficient of x)

$ab = 8$ (constant term)

Thus given polynomial can be written as $x^2 + 2x + 4x + 8$

$= (x^2 + 2x) + (4x + 8)$

[Here 4 terms obtained in 2^{nd} step have been written as sum of two groups].

$= (x + 2)(x + 4)$

Again x + 2 which is common in both terms, has been taken out

or

$x^2 + 6x + 8$

Here $a = 4$ and $b = 2$

$x^2 + (4 + 2)x + (x \times 2)$

$\Rightarrow$ $(x + 4)(x + 2)$ (Using Identity $x^2 + (a + b)x + ab = (x + a)(x + b)$