DIRECT AND INVERSE PROPORTIONS

TIME AND WORK

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We use the principles of direct and indirect variations to solve problems on 'time and work', such as :

"More men do more work and less men do less work" (Direct variation)

"More men take less time to do a work and less men take more time to do the same work." (Indirect variation)

The problems on "time and work" are divided in two categories:

(i) To find the work done in a given period of time.

(ii) To find the time required to complete a given job.

Working Rules

We shall use the unitary method by considering the following fundamental rules for solving problems regarding time and work:

(i) A complete job or work is taken to be one.

(ii) Time to complete a work $= \frac{\text{Total work to be done}}{\text{Part of the work done in one day}}$.

Ex.1 Ratan takes 5 days to complete a certain job and shankar takes 8 days to do the same job. If both of them work together, how long will they take to complete the work?

Sol. Since, Ratan takes 5 days to complete the given work

$\therefore$ Ratan finishes $\frac{1}{5}$ part in 1 day.

Similarly, Shankar takes 8 days to complete the work.

Therefore, Shankar finishes $\frac{1}{8}$ part in 1 day.

$\therefore$ In a day, they together will finish $= \frac{1}{5} + \frac{1}{8} = \frac{8+5}{40} = \frac{13}{40}$

i.e., $\frac{13}{40}$ part of the work.

So, they both will take $\frac{40}{13}$ days $3\frac{1}{13}$ days to complete the work. Hence, the complete work will be finished by them together in $3\frac{1}{13}$ days.

Ex.2 Kshitij can do a piece of work in 20 days and Rohan can do the same work in 15 days. They work together for 5 days and then Rohan leaves. In how many days will Kshitij alone finish the remaining work?

Sol. Since, Kshitij completes the work in 20 days

$\therefore$ Kshitij's 1 day work $= \frac{1}{20}$ part

Now, Rohan completes the work in 15 days.

Similarly, Rohan's 1 day work $= \frac{1}{15}$ part

$\therefore$ Their combined work for 1 day $= \frac{1}{20} + \frac{1}{15} = \frac{3+4}{60} = \frac{7}{60}$

$\therefore$ Their combined work for 5 days $= 5 \times \frac{7}{60} = \frac{7}{12}$ part

Remaining work = Complete work – Work done in 5 days

$= 1 - \frac{7}{12}$

$= \frac{12-7}{12} = \frac{5}{12}$ part

Now, the remaining work is to be completed by Kshitij alone.

Kshitij can complete the whole work in 20 days.

So, he will complete $\frac{5}{12}$ work in

$\left(\frac{5}{12} \times 20\right)$ days, i.e., $\frac{25}{3}$ days or $8\frac{1}{3}$ days.

Ex.3 A and B can do a piece of work in 10 days; B and C in 15 days; C and A in 12 days. How long would A and B take separately to do the same work ?

Sol. A and B can complete the work in 10 days.

$\therefore$ (A and B)'s one day work $= \frac{1}{10}$ part

Similarly,

(B and C)'s one day work $= \frac{1}{15}$ part

(C and A)'s one day work $= \frac{1}{12}$ part

Adding up, we get

2(A and B and C)'s work in 1 day

$= \left(\frac{1}{10}+\frac{1}{15}+\frac{1}{12}\right)$ part

$= \frac{6+4+5}{60} = \frac{15}{60} = \frac{1}{4}$ part

$\therefore$ (A and B and C) can do in 1 day $= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ part

Now,

Part of work A can do in 1 day

= (1 day work of A and B and C) – (1 day work of B and C)

$= \left(\frac{1}{8}\right) - \left(\frac{1}{15}\right)$

$= \frac{15-8}{120} = \frac{7}{120}$ part

Hence, A can complete the work in $\left(1 \times \frac{120}{7}\right)$ days, i.e., $\frac{120}{7}$ or $17\frac{1}{7}$ days.

Similarly,

Part of the work B can do in 1 day

= (1 day work of A and B and C) – (1 day work of A and C)

$= \left(\frac{1}{8}\right) - \left(\frac{1}{12}\right) = \frac{3-2}{24} = \frac{1}{24}$

Hence, B can complete the work in $\left(1 \times \frac{24}{1}\right)$ days, i.e., 24 days.

Ex.4 A contractor undertakes to construct a road in 20 days and engages 12 workers. After 16 days, he finds that only $\frac{2}{3}$ part of the work has been done. How many more workers should he now engage in order to finish the job in time?

Sol. From the question, it is clear that $\frac{2}{3}$ part of the work has been completed by 12 workers in 16 days.

$\therefore$ Remaining work $= 1 - \frac{2}{3} = \frac{1}{3}$

Remaining number of days $= 20 - 16 = 4$

Thus, $\frac{1}{3}$ part of the work is to be finished in 4 days.

$\therefore$ Number of workers required to complete $\frac{2}{3}$ part of work in 16 days $= 12$

Number of workers required to complete 1 work in 16 days $= 12 \times \frac{3}{2} \times 16$

Number of workers required to complete $\frac{1}{3}$ work in 1 day $= 12 \times \frac{3}{2} \times 16 \times \frac{1}{3}$

Number of workers required to complete $\frac{1}{3}$ work in 4 days $= 12 \times \frac{3}{2} \times 16 \times \frac{1}{3} \times \frac{1}{4}$

$\therefore$ Number of additional workers required $= 24 - 12 = 12$

Hence, the contractor will have to engage 12 more workers to complete the work in time.

Ex.5 A garrison of 350 men had food for 25 days. However, after 5 days a reinforcement of 150 men join them. How long will the food last now?

Sol. As 350 men have already eaten the food for 5 days, so they will eat the remaining food in 20 days. Since 150 men have arrived, the number of men now becomes 500. Thus, it can be represented in a tabular form as,

Men	350	500
Number of days	20	x

Clearly, it is the case of inverse proportion.

Thus, ratio of men = inverse ratio of number of days.

or $\frac{350}{500} = \frac{x}{20}$ or $x = \frac{350 \times 20}{500} = 14$

$\therefore$ The food will last for 14 days.

Time and Work

The amount of work done by a person varies directly with the time taken by him or her.

If a man completes a work in 20 days, thus by unitary method we can say that he will complete $\frac{1}{20}$ th of the work in one day.

Rule 1.

If A completes a work in n days, then the work done by A in one day $= \frac{1}{n}$ th part of the works.

Rule 2.

If A completes $\frac{1}{n}$ th part of the work in one day, then A will take n days to complete the work.

Ex.6 Ashish takes 12 days to do a piece of work, while Arjun takes 15 days to do the work. Find the time taken by them if they work together.

Sol. Ashish takes 12 days to do piece of work.

$\therefore$ In one day he does $\frac{1}{12}$ th of the work.

Arjun takes 15 days to do a piece of work.

$\therefore$ In one day he does $\frac{1}{15}$ th of the work.

$\therefore$ Together they do $\left(\frac{1}{12}+\frac{1}{15}\right)$ th of the work in one day.

i.e. $\frac{1}{12}+\frac{1}{15}=\frac{5+4}{60}=\frac{9}{60}=\frac{3}{20}$

$\therefore$ In one day they will finish $\frac{3}{20}$ th of the work

$\therefore$ They take $\frac{20}{3}=6\frac{2}{3}$ days to finish the work.

Ex.7 Two taps take 12 hours and 16 hours respectively to fill a tank. Find the time taken to fill the tank if they are open at the same time.

Sol. Time taken by first pipe = 12 hours

$\therefore$ In 1 hour it fills $\frac{1}{12}$ th of the tank.

Time taken by second pipe = 16 hours

$\therefore$ In 1 hour it fills $\frac{1}{16}$ th of the tank.

$\therefore$ Total work done in 1 hours = $\frac{1}{12}+\frac{1}{16}=\frac{4+3}{48}=\frac{7}{48}$

$\therefore$ Time taken = $\frac{48}{7}$ hour = 6 hours 51 minutes (approximately).

Ex.8 Mohinder ploughs a field in 6 days and Ram ploughs the same field in 12 days. How long both of them take to plough the same field working together ?

Sol. Mohinder ploughs in 6 days = 1 field

Mohinder ploughs in 1 day = $\frac{1}{6}$ th field

Ram ploughs in 1 day = $\frac{1}{12}$ th field

Both Ram and Mohinder ploughs in

1 day = $\left(\frac{1}{6}+\frac{1}{12}\right)$ th field.

$= \frac{2+1}{12} = \frac{3}{12} = \frac{1}{4}$ field.

Now $\frac{1}{4}$ th of the field is ploughed by them in 1 day.

$\therefore$ The complete field will be ploughed by them in $1 \times \frac{4}{1} = 4$ days.

Ex.9 12 men working 8 hours a day complete a work in 10 days. How long would 16 men working $7\frac{1}{2}$ hours a day take to complete the same work ?

Sol. Let the work completed in x days.

Men	Hours	Days
12	8	10
16	$\frac{15}{2}$	x

More men less time
Less men more time
Thus, it is inverse variation

$\left.\begin{matrix} 16:12 \\ \frac{15}{2}:8 \end{matrix}\right\} :: 10 : x$

$\therefore x = \frac{10 \times 12 \times 8 \times 2}{16 \times 15} = 8$

$\therefore$ 16 men will complete the same work in 8 days.

Ex.10 2 men and 3 boys can harvest a field in 7 days. How long would 1 man and 2 boys take to harvest the same field?

Sol. Given that 2 men and 3 boys harvest a field in 7 days. Thus, let us calculate the amount of field harvested by each one in one day.

2 men harvest 1 field in 7 days.

In one day 2 men will harvest $\frac{1}{7}$ th of the field.

In one day 1 man will harvest $\frac{1}{2 \times 7}$ th, i.e. $\frac{1}{14}$ th of the field.

Similarly, 1 boy will harvest $\frac{1}{3\times 7}$ th,

i.e. $\frac{1}{21}$ th of the field in one day.

Now, we have to find the time taken by 1 man and 2 boys to harvest the field. Adding the amounts of work completed by 1 man and 2 boys in one day, we get

$\frac{1}{14} + \frac{2}{21} = \frac{3+4}{42} = \frac{7}{42}$ or $\frac{1}{6}$

Thus, they will take 6 days to complete the harvesting.