DIRECT AND INVERSE PROPORTIONS

INVERSE VARIATION

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Consider the following table showing various number of men and the corresponding number of days to complete the work.

x (No. of men)	40	20	10	8	5	1
y (No. of days)	1	2	4	5	8	40

Here, the number of men are denoted by x and the corresponding number of days by y.

In this case, when the number of men increases, the corresponding number of days decreases. But, by a careful observation, we find that the product of the corresponding number of men and days is always the same :

$40 \times 1 = 40$

$20 \times 2 = 40$

$10 \times 4 = 40$

$8 \times 5 = 40$

$5 \times 8 = 40$

$1 \times 40 = 40$

That is the product (40) is constant.

In general, it may be expressed as

$xy = k$(constant)

Let x_1 and x_2 be two values of x and their corresponding values of y be $y_1$and y_2.

Then, $x_1y_1 = k$ and $x_2y_2 = k$

$\therefore \quad \frac{x_1y_1}{x_2y_2} = \frac{k}{k} = 1$

or $x_1y_1 = x_2y_2$ or $\frac{x_1}{x_2} = \frac{y_2}{y_1}$

Hence, we conclude that, if two quantities x and y vary such that their product xy remains constant, then we say that x and y vary inversely and the variation is called inverse variation.

The relation $\frac{x_1}{x_2} = \frac{y_2}{y_1}$ is used to find the value of any one of x_1, x_2, y_1 and y_2, if the other three are known.

Ex.1 In a boarding house of 80 boys, there is food provisions for 30 days. If 20 more boys join the boarding house, how long will the provisions last?

Sol. Obviously, more the boys the sooner would the provisions exhaust. It is, therefore, the case of inverse variation. The number of boys in the two situations are :

80 and (80 + 20), i.e., 100 respectively. If the provisions last for x days when the number of boys increased from 80 to 100, we can have the following table :

Number of Boys	Number of Days
80	30
100	x

Here, the ratio between the like terms are :

$\frac{80}{100}$ and $\frac{30}{x}$

Since, the problem is of inverse variation, we will invert the ratio and then equate them :

$\frac{x}{30} = \frac{80}{100}$

or $\frac{x}{30} = \frac{4}{5}$

or $x = \frac{4\times30}{5} = \frac{4\times6}{1}$

or $x = 24$

Therefore, the provisions will last for 24 days.

Ex.2 A jeep finishes a journey in 9 hours at a speed of 60 km per hour. by how much should its speed be increased so that it may take only 6 hours to finish the same journey?

Sol. Let the desired speed of the jeep be x km per hour, then we have :

Number of Hours	Speed of the Jeep (in km per hour)
9	60
6	x

Since, the greater the speed, the lesser the time taken. Therefore, the number of hours and speed vary inversely.

$\therefore \quad \frac{9}{6} = \frac{x}{60}$

or $\frac{x}{60} = \frac{9}{6}$

or $x = \frac{9}{6} \times 60 = \frac{9 \times 10}{1} = 90$

$\therefore$ Increase in speed = (90 – 60) km per hour = 30 km per hour

Thus, the required increase in speed is 30 km per hour.

Problems on Time and Distance

The speed of a moving body is the distance moved in unit time. It is usually represented either in km/h or m/s.

Relation among Speed, Time and Distance

The relation among speed, distance and time is given by Distance covered = Speed × Time taken. If any two of them are given, it is easy to determine the third one. The above relation can also be expressed in the following manners:

$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

or $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

We talk about speed, say 27 km/h, it means that we are actually talking about its average speed. By average speed of a vehicle, we mean that constant speed at which the vehicle would cover a distance of 27 km in an hour. Unless mentioned otherwise, by speed we shall mean an average speed.

Ex.3 A man takes 12 hours to travel 48 kilometres. How long will he take to travel 72 kilometres?

Sol. Since the man travels 48 km in 12 hours, therefore, one kilometre is travelled in $\frac{12}{48}$ hours.

He travels 72 km in $\frac{12\times 72}{48}$ hours or in 18 hours.

Ex.4 A train of 320 metres length, is running at a speed of 72 km/h. How much time will it take to cross a pole ?

Sol. Speed of the train $= 72$ km/h

$= 72 \times 1000$ m/h

$= \frac{72000}{60\times 60}$ m/s $= 20$ m/s

Length of the train $= 320$ m

Since the train of length 320 m has to cross the pole of negligible dimension, it has to cross the length of itself, i.e., 320 m.

Thus, distance to be covered $= 320$ m

Now, using the relation time $= \frac{\text{Distance}}{\text{Speed}}$, we get the required time for the train to cross a distance of 320 m $= \frac{320}{20}$ [$\because$ Speed of the train is 20 m/s (found above)]

Hence, the train takes 16 seconds to cross the pole.