EXPONENTS AND POWERS

LAWS OF EXPONENTS

LAWS OF EXPONENTS (POSITIVE EXPONENTS)

There are certain laws that govern the operations in numbers which are expressed in the exponential notation.

Law-I If x is a rational number and m and n are positive integers, then $x^m \times x^n = x^{m+n}$.

For Example, $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^3$

$$= \left(\frac{2}{3}\times\frac{2}{3}\right)\times\left(\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\right) = \left(\frac{2}{3}\right)^{2+3} = \left(\frac{2}{3}\right)^5$$

Law-II If x is a rational number and m and n are positive integers, such that m > n, then $x^m \div x^n = x^{m-n}$

For Example, $\left(\frac{4}{5}\right)^5 \div \left(\frac{4}{5}\right)^3 = \left(\frac{4}{5}\right)^5 \times \left(\frac{5}{4}\right)^3$

$$= \frac{4}{5}\times\frac{4}{5}\times\frac{4}{5}\times\frac{4}{5}\times\frac{4}{5}\times\frac{5}{4}\times\frac{5}{4}\times\frac{5}{4}$$

$$= \frac{4}{5}\times\frac{4}{5} = \left(\frac{4}{5}\right)^2 \text{ or } \left(\frac{4}{5}\right)^{5-3}$$

Law-III If x is a rational number and m and n are positive integers, then $(x^m)^n = x^{m \times n} = x^{mn}$.

For Example, $(2^2)^3 = 2^2 \times 2^2 \times 2^2 = 2^{2+2+2}$

$= 2^6 = 2^{2\times 3}$ (From Law I)

Law-IV If $\frac{x}{y}$ is any rational number and *m* is any positive integer, then $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$.

For Example, $\left(\frac{2}{7}\right)^3 = \frac{2}{7}\times\frac{2}{7}\times\frac{2}{7} = \frac{2^3}{7^3}$

Law-V

1. **The Zero Exponent**: If x is a rational number and $x \neq 0$, then $x^0 = 1$.

For example, $5^0 = 1$ and $(-7)^0 = 1$.

2. **The Negative Exponent**:

If x is a rational number different from 0, then x^{-1} denotes the reciprocal of x.

We know that the reciprocal of x is $\frac{1}{x}$. Therefore, $x^{-1} = \frac{1}{x}$.

For Example, $6^{-1} = \frac{1}{6}$ and $11^{-1} = \frac{1}{11}$.

Law VI If x is any rational number different from 0 and m is a positive integer, then x^{-m} denotes the reciprocal of x^m.

Also, $\left(\frac{x}{y}\right)^{-m} = \left(\frac{y}{x}\right)^{m}$.

For Example, $\left(\frac{2}{3}\right)^{-3} = \frac{1}{\left(\frac{2}{3}\right)^3}$

$$= \frac{1}{\frac{2^3}{3^3}} = \frac{3^3}{2^3} = \left(\frac{3}{2}\right)^3$$

Ex.1 Find the value of $\left(\frac{-3}{4}\right)^3 \times \left(\frac{-3}{4}\right)^4$.

Sol. The numbers $\left(\frac{-3}{4}\right)^3$ and $\left(\frac{-3}{4}\right)^4$ have the same bases. So to find the product, we add their powers.

Thus, $\left(\frac{-3}{4}\right)^3 \times \left(\frac{-3}{4}\right)^4 = \left(\frac{-3}{4}\right)^{3+4} = \left(\frac{-3}{4}\right)^7$

$$= \frac{-2187}{16384}$$

Ex.2 Evaluate : $\left(\frac{3}{4}\right)^6 \div \left(\frac{3}{4}\right)^4$

Sol. The numbers $\left(\frac{3}{4}\right)^6$ and $\left(\frac{3}{4}\right)^4$ have the same bases. So, to find the solution, we have to subtract the powers.

$$\therefore \left(\frac{3}{4}\right)^6 \div \left(\frac{3}{4}\right)^4 = \left(\frac{3}{4}\right)^{6-4} = \left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$$

Ex.3 Simplify :

(a) $\left(\frac{2}{3}\right)^{-3} \times \left(\frac{2}{3}\right)^{-1}$ (b) $\left[\left(\frac{-1}{2}\right)^{-3}\right]^{-2}$

Sol. (a) The given product can be written as -

$$\left(\frac{2}{3}\right)^{-3} \times \left(\frac{2}{3}\right)^{-1} = \left(\frac{2}{3}\right)^{-3+(-1)}$$

$$= \left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^{4} = \frac{81}{16}$$

(b) The exponent $\left[\left(\frac{-1}{2}\right)^{-3}\right]^{-2}$ can be written as

$$\left(\frac{-1}{2}\right)^{[(-3)\times(-2)]} = \left(\frac{-1}{2}\right)^{6} = \frac{1}{64}$$

Ex.4 If $\frac{p}{q} = \left(\frac{3}{2}\right)^{-2} \div \left(\frac{6}{7}\right)^0$, find the value of $\left(\frac{p}{q}\right)^{-3}$.

Sol. We have $\left(\frac{p}{q}\right) = \left(\frac{3}{2}\right)^{-2} \div \left(\frac{6}{7}\right)^0 = \left(\frac{2}{3}\right)^2 \div 1$

$$= \left(\frac{2}{3}\right)^2 \times 1 = \left(\frac{2}{3}\right)^2$$

$$\therefore \quad \left(\frac{p}{q}\right)^{-3} = \left[\left(\frac{2}{3}\right)^{2}\right]^{-3} = \left(\frac{2}{3}\right)^{-6} = \left(\frac{3}{2}\right)^{6}$$

Ex.5 If $5^{2x+1} \div 25 = 125$, find the value of x.

Sol. We have $5^{2x+1} \div 25 = 125$

or $\dfrac{5^{2x+1}}{25} = 125$ or $\dfrac{5^{2x+1}}{5 \times 5} = 5 \times 5 \times 5$

or $\dfrac{5^{2x+1}}{5^2} = 5^3$ or $5^{2x+1-2} = 5^3$

or $5^{2x-1} = 5^3$ or $2x - 1 = 3$

or $2x = 4$ or $x = 2$

Ex.6 Simplify and express the result as a power of 2.

(a) $(3^4 \times 3^5) \div 3^9$ (b) $\left[\left(\frac{1}{5}\right)^{6} \div \left(\frac{1}{5}\right)^{5}\right] \div \frac{1}{5}$

Sol. (a) We have $(3^4 \times 3^5) \div 3^9 = 3^{4+5} \div 3^9 = 3^9 \div 3^9$

$= 3^{9-9} = 3^0 = 1 = 2^0$,

Where 1 can be expressed as a power of 2 as 2^0.

(b) $$\left[\left(\frac{1}{5}\right)^{6} \div \left(\frac{1}{5}\right)^{5}\right] \div \frac{1}{5} = \left[\frac{\left(\frac{1}{5}\right)^{6}}{\left(\frac{1}{5}\right)^{5}}\right] \div \frac{1}{5}$$

$$= \left[\left(\frac{1}{5}\right)^{6-5}\right] \div \frac{1}{5}$$

$$= \left(\frac{1}{5}\right)^{1} \div \left(\frac{1}{5}\right)^{1} = \left(\frac{1}{5}\right)^{1-1} = \left(\frac{1}{5}\right)^{0} = 1 = 2^0$$

Again, 1 can be expressed as the power of 2 as 2^0.

Ex.7 By what number should we multiply 7^{-5}, so that the product may be equal to 7 ?

Sol. Let the number be x.

Then $7^{-5} \times x = 7$

or $x = \frac{7}{7^{-5}} = 7 \times 7^5 = 7^6$

$\therefore$ The number is 7^6.

Ex.8 By what number should 7^5 be divided, so that the quotient is 7^{-3} ?

Sol. Let the number be x.

Then $7^5 \div x = 7^{-3}$

Or $\frac{7^5}{x} = 7^{-3}$ or $\frac{7^5}{7^{-3}} = x$

Or $x = 7^5 \times 7^3 = 7^8$

$\therefore$ The number is 7^8.