MENSURATION

AREA OF PARALLELOGRAM

AREA OF QUADRILATERALS

Area of a Parallelogram:

Consider parallelogram ABCD.

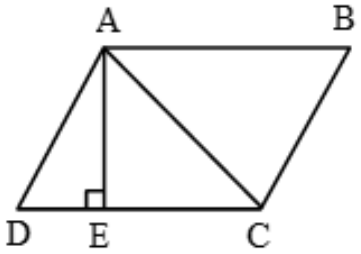
Let AC be a diagonal

In ΔADC and ΔCBA

$AD = CB$, $CD = AB$

AC is common

$\therefore \ \Delta ADC \cong \Delta CBA$



$\therefore$ Area of parallelogram ABCD

= Area of ΔADC + Area of ΔABC

$= 2 \times$ Area of ΔADC

$= 2 \times (\frac{1}{2} CD \times AE)$ (where $AE \perp DC$)

$= DC \times AE$

i.e. Area of parallelogram = base × height

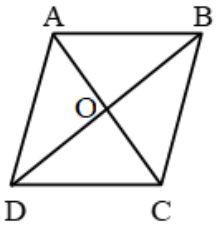
Area of a Rhombus

Since a rhombus is also a parallelogram, its area is given by

Area of rhombus = base × height

The area of a rhombus can also be found if the length of the diagonals are given. Let ABCD

be a rhombus. We know that its diagonals AC and BD bisect each other at right angles.



Area of rhombus ABCD = area of ΔABD + area of ΔCBD

$= ½ (BD \times AO) + ½(BD \times CO)$

(since $AO \perp BD$ and $CO \perp BD$)

$= ½ BD (AO + CO) = ½ BD \times AC$

i.e. Area of rhombus $= ½ \times$ product of diagonals

Area of a Trapezium :

Let ABCD be a trapezium with AB || DC. Draw AE and BF perpendicular to DC.

Then $AE = BF =$ height of trapezium $= h$

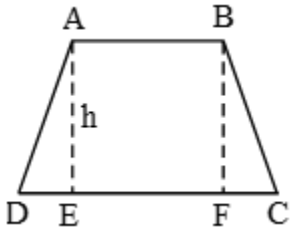
Area of trapezium ABCD = Area of ΔADE + Area of rectangle ABFE + Area of ΔBCF

$= ½ \times DE \times h + EF \times h + ½ FC \times h$

$= ½ h (DE + 2EF + FC)$

$= ½ h (DE + EF + FC + EF)$

$= ½ h (DC + AB)$ (since $EF = AB$)



i.e. Area of trapezium $= ½ \times$ (sum of parallel sides) $\times$ (distance between parallel sides)

Area of a Quadrilateral :

Let ABCD be a quadrilateral, and AC be one of its diagonals. Draw perpendiculars BE and DF from B and D respectively to AC.

Area of quadrilateral ABCD

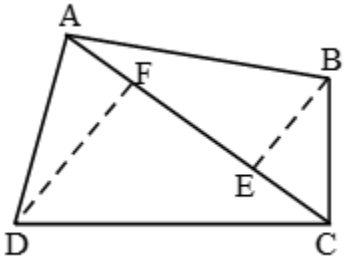
= Area of ΔABC + Area of ΔADC

$= ½ AC \times BE + ½ AC \times DF$

$= ½ AC (BE + DF)$

If $AC = d$, $BE = h_1$ and $DF = h_2$ then

Area of quadrilateral $= ½d (h_1 + h_2)$



Ex.1 A rectangle and a parallelogram have the same area of 72 cm^2. The breadth of the rectangle is 8 cm. The height of the parallelogram is 9 cm. Find the base of the parallelogram and the length of the rectangle.

Sol. Area of rectangle $= \lambda \times b = \lambda \times 8 = 72$

$\therefore \quad \lambda = 9$ cm

Area of parallelogram = base × height

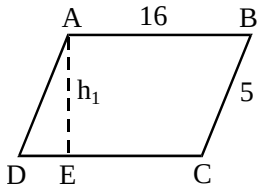
= base × 9 = 72

$\therefore$ Base = 8 cm

Ex.2 The area of a parallelogram is 64 cm^2. Its sides are 16 cm and 5 cm. Find the two heights of the parallelogram.

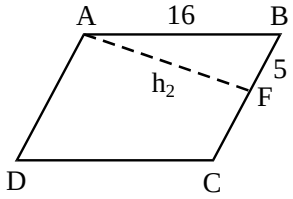
Sol. (i) Area = base × height $= 16 \times h_1 = 64$

$\therefore \quad h_1 = 4$cm



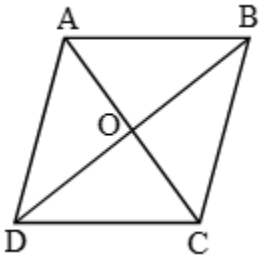
(ii) Area = base × height $= 5 \times h_2 = 64$

$\therefore \quad h_2 = 12.8$ cm



Ex.3 The diagonals of a rhombus measure 10 cm and 24 cm. Find its area. Also find the measure of its side.

Sol. AC = 10 cm, BD = 24 cm



Area = ½ $(d_1 \times d_2)$ = ½ × 10 × 24 cm^2 = 120 cm^2

In ΔAOB, $\angle AOB = 90^\circ$, AO = ½ AC = 5 cm,

BO = ½ BD = 12 cm.

$\therefore AB^2 = AO^2 + OB^2 = 25 + 144 = 169 = 13 \times 13$

$\therefore$ AB = 13 cm

$\therefore$ Measure of λ side = 13 cm

Ex.4 In rhombus ABCD, AB = 7.5 cm, and AC = 12 cm. Find the area of the rhombus.

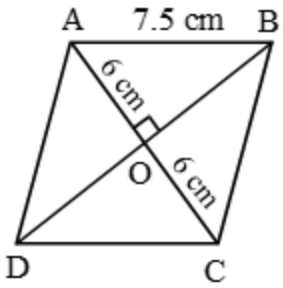
Sol. In ΔABO, $\angle$AOB = 90º, AO = ½ AC = 6 cm, AB = 7.5 cm

$\therefore$ $OB^2 = AB^2 - OA^2$

$= (7.5)^2 - 6^2 = 56.25 - 36 = 20.25$

$\therefore$ OB = $\sqrt{2025} = 4.5$ cm

$\therefore$ BD = 2 × OB = 9 cm



Area of rhombus = ½ $d_1 \times d_2$

$= ½ \times 9 \times 12$ cm^2 = 54cm^2

Ex.5 In the trapezium PQRS, $\angle P = \angle S = 90º$, PQ = QR = 13 cm, PS = 12 cm and SR = 18 cm. Find the area of the trapezium.

Sol. The parallel sides are PQ and SR, and the distance between them is PS,

since $\angle P = \angle S = 90º$

$\therefore$ Area = ½ × sum of parallel sides × heights

$= ½ \times (13 + 18) \times 12$ cm^2

= 186 cm^2