RATIONAL NUMBERS

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If x and y are two rational number. Such that x < y then ½(x +y) is a rational number lying between x and y.

Ex.1 Give three rational numbers lying between $\frac{1}{3}$ and $\frac{1}{2}$.

Sol. The rational number $= \frac{1}{2}\times\left(\frac{1}{3}+\frac{1}{2}\right)$ lies between $\frac{1}{3}$ and $\frac{1}{2}$

Now, $\frac{1}{2}\times\left(\frac{1}{3}+\frac{1}{2}\right)=\frac{1}{2}\times\left(\frac{2+3}{6}\right)=\frac{1}{2}\times\frac{5}{6}=\frac{5}{12}$

Therefore, $\frac{1}{3}<\frac{5}{12}<\frac{1}{2}$.

Let us now find a rational number between $\frac{1}{3}$ and $\frac{5}{12}$

We know that $\frac{1}{2}\left(\frac{1}{3}+\frac{5}{12}\right)$ is one such mmber

Also, $\frac{1}{2}\left(\frac{1}{3}+\frac{5}{12}\right)=\frac{1}{2}\left(\frac{4}{12}+\frac{5}{12}\right)=\frac{1}{2}\times\frac{9}{12}=\frac{9}{24}=\frac{3}{8}$

$\frac{1}{3}<\frac{3}{8}<\frac{5}{12}<\frac{1}{2}$

Now let us find a rational number between $\frac{5}{12}$ and $\frac{1}{2}$

One such number is

$$\frac{1}{2}\left(\frac{5}{12}+\frac{1}{2}\right)=\frac{1}{2}\left(\frac{5}{12}+\frac{6}{12}\right)=\frac{1}{2}\times\frac{11}{12}=\frac{11}{24}$$

$$\frac{1}{3}<\frac{3}{8}<\frac{5}{12}<\frac{11}{24}<\frac{1}{2}$$

Hence, $\frac{3}{8},\frac{5}{12},\frac{11}{24}$ are the required three rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$.

Ex.2 Write any 3 rational numbers between –2 and 0.

Sol. –2 can be written as $\frac{-20}{10}$ and 0 as $\frac{0}{10}$.

Thus we have $\frac{-19}{10}, \frac{-18}{10}, \frac{-17}{10}, \frac{-16}{10}, \frac{-15}{10}, \ldots\ldots \frac{-1}{10}$ between – 2 and 0.

Ex.3 Find any ten rational numbers between $\frac{-5}{6}$ and $\frac{5}{8}$.

Sol. We first convert $\frac{-5}{6}$ and $\frac{5}{8}$ to rational numbers with the same denominators.

$$\frac{-5\times4}{6\times4} = \frac{-20}{24} \text{ and } \frac{5\times3}{8\times3} = \frac{15}{24}.$$

Thus we have, $\frac{-19}{24}, \frac{-18}{24}, \frac{-17}{24}, \ldots\ldots \frac{14}{24}$ as the rational numbers between $\frac{-20}{44}$ and $\frac{15}{24}$

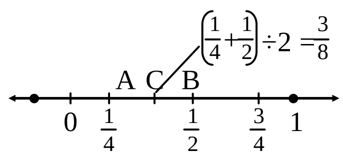
Ex.4 Find a rational number between $\frac{1}{4}$ and $\frac{1}{2}$.

Sol. We find the mean of the given rational numbers.

$$\left(\frac{1}{4}+\frac{1}{2}\right)\div 2 = \left(\frac{1+2}{4}\right)\div 2 = \frac{3}{4}\times\frac{1}{2} = \frac{3}{8}$$

$\frac{3}{8}$ lies between $\frac{1}{4}$ and $\frac{1}{2}$.

This can be seen on the number line also.



We find mid point of AB which is C, represented by $\left(\frac{1}{4}+\frac{1}{2}\right)\div 2 = \frac{3}{8}$.

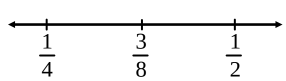
We find that $\frac{1}{4} < \frac{3}{8} < \frac{1}{2}$.

If a and b are two rational numbers, then $\frac{a+b}{2}$ is a rational number between a and b such that $a < \frac{a+b}{2} < b$.

This again shows that there are countless number of rational numbers between any two given rational numbers.

Ex.5 Find three rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.

Sol. We find the mean of the given rational number.

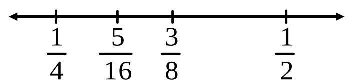


As given in the above example, the mean is $\frac{3}{8}$ and $\frac{1}{4} < \frac{3}{8} < \frac{1}{2}$.

Now we find another rational number between $\frac{1}{4}$ and $\frac{3}{8}$. For this, we again find the mean of $\frac{1}{4}$ and $\frac{3}{8}$. That is,

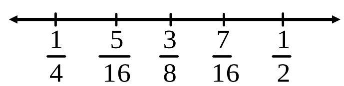
$$\left(\frac{1}{4}+\frac{3}{8}\right) \div 2 = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16}$$

$$\frac{1}{4} < \frac{5}{16} < \frac{3}{8} < \frac{1}{2}$$

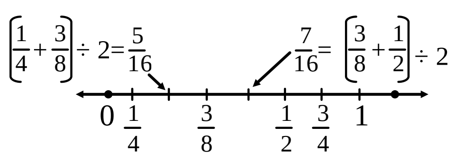


Now find the mean of $\frac{3}{8}$ and $\frac{1}{2}$. We have, $\left(\frac{3}{8}+\frac{1}{2}\right) \div 2 = \frac{7}{8} \times \frac{1}{2} = \frac{7}{16}$

Thus we get $\frac{1}{4} < \frac{5}{16} < \frac{3}{8} < \frac{7}{16} < \frac{1}{2}$.



Thus, $\frac{5}{16}, \frac{3}{8}, \frac{7}{16}$ are the three rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$. This can clearly be shown on the number line as follows :



In the same way we can obtain as many rational numbers as we want between two given rational numbers. You have noticed that there are countless rational numbers between any two given rational numbers.