RATIONAL NUMBER

POWERS

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Exponential Notation and Rational Numbers :

Exponential notation can be extended to rational numbers. For example: $\left(\frac{4}{5}\right)\times\left(\frac{4}{5}\right)\times\left(\frac{4}{5}\right)$ can be written as $\left(\frac{4}{5}\right)^3$ which is read as $\frac{4}{5}$ raised to the power 3.

(i) $\left(\frac{3}{4}\right)^3=\left(\frac{3}{4}\right)\times\left(\frac{3}{4}\right)\times\left(\frac{3}{4}\right)=\frac{3^3}{4^3}=\frac{27}{64}$

(ii) $\left(\frac{-5}{6}\right)^2=\left(\frac{-5}{6}\right)\times\left(\frac{-5}{6}\right)=\frac{(-5)^2}{6^2}=\frac{25}{36}$

(ii) $\left(\frac{-2}{3}\right)^3=\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)=\frac{(-2)^3}{3^3}=\frac{-8}{27}$

In general, if $\frac{x}{y}$ is a rational number and a is a positive integer, then

$$\left(\frac{x}{y}\right)^a=\frac{x^a}{y^a}$$

Ex.1 Evaluate $\left(-\frac{4}{5}\right)^3$.

Sol. $\left(-\frac{4}{5}\right)^3=\left(-\frac{4}{5}\right)\times\left(-\frac{4}{5}\right)\times\left(-\frac{4}{5}\right)=\frac{(-4)^3}{5^3}$

$=\frac{-64}{125}$

Ex.2 Express $\frac{27}{64}$ and $\frac{-8}{27}$ as the powers of rational numbers.

Sol. $\frac{27}{64} = \frac{3\times3\times3}{4\times4\times4} = \frac{3^3}{4^3} = \left(\frac{3}{4}\right)^3$

and $\frac{-8}{27} = \frac{(-2)\times(-2)\times(-2)}{3\times3\times3} = \frac{(-2)^3}{3^3} = \left(\frac{-2}{3}\right)^3$

Reciprocals with Positive Integral Exponents:

The reciprocal of 2 is $\frac{1}{2}$, reciprocal of 2^3 is $\frac{1}{2^3}$.

Reciprocal of $\left(\frac{2}{3}\right)^4 = \frac{1}{\left(\frac{2}{3}\right)^4} = \frac{1}{\frac{2^4}{3^4}} = \frac{3^4}{2^4} = \left(\frac{3}{2}\right)^4$

Reciprocal of $\left(\frac{-4}{5}\right)^4 = \left(\frac{-5}{4}\right)^4$ and

Reciprocal of $\left(\frac{1}{3}\right)^5 = \left(\frac{3}{1}\right)^5 = 3^5$

Reciprocals with Negative Integral Exponents

Reciprocal of $2 = \frac{1}{2} = \frac{1}{2^1}$.

Therefore, the reciprocal of 2 is 2^{-1}. The reciprocal of $3^2 = \frac{1}{3^2} = 3^{-2}$.

Reciprocal of $\left(\frac{4}{5}\right)^2 = \left(\frac{4}{5}\right)^{-2}$

Reciprocal of $\left(\frac{-2}{3}\right)^3 = \left(\frac{-2}{3}\right)^{-3}$, etc.

In general, if x is any rational number other than zero and a is any positive integer, then:

$$x^{-a} = \frac{1}{x^a}$$

Ex.3 Simplify $\left(\frac{2}{3}\right)^{-3} \div \left(\frac{4}{3}\right)^{-2}$.

Sol. $\left(\frac{2}{3}\right)^{-3} \div \left(\frac{4}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{3} \div \left(\frac{3}{4}\right)^{2}$

$$= \frac{3\times3\times3}{2\times2\times2} \div \frac{3\times3}{4\times4}$$

$$= \frac{27}{8} \div \frac{9}{16} = \frac{27}{8} \times \frac{16}{9} = 6$$

Laws of Exponents :

1. Consider the following.

(i) $3^3 \times 3^4 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$$= 3^7 = 3^{3+4}$$

(ii) $\left(\frac{5}{2}\right)^2 \times \left(\frac{5}{2}\right)^3 = \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2}$

$$= \left(\frac{5}{2}\right)^5 = \left(\frac{5}{2}\right)^{2+3}$$

$$x^a \times x^b = x^{a+b}$$

2. (i) $2^5 \div 2^2 = \frac{2\times2\times2\times2\times2}{2\times2} = 2 \times 2 \times 2$

$$= 2^3 = 2^{5-2}$$

(ii) $\left(\frac{2}{3}\right)^6 \div \left(\frac{2}{3}\right)^2 = \dfrac{\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}}{\frac{2}{3}\times\frac{2}{3}}$

$= \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{6-2}$

$x^a \div x^b = x^{a-b}$

3. (i) $(2^3)^2 = (2 \times 2 \times 2)^2$

$= (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$= 2^6 = 2^3 \times 2^3$

(ii) $\left\{\left(\frac{2}{3}\right)^3\right\}^2 = \left(\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\right)^2$

$= \left(\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\right) \times \left(\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\right) = \left(\frac{2}{3}\right)^6 = \left(\frac{2}{3}\right)^{3\times 2}$

$(x^a)^b = x^{ab}$

4. (i) $2^4 \times 3^4 = (2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3 \times 3)$

$= (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) = (2 \times 3)^4$

(ii) $\left(\frac{3}{5}\right)^4 \times \left(\frac{1}{2}\right)^4$

$= \left(\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\right) \times \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$

$= \left(\frac{3}{5}\times\frac{1}{2}\right) \times \left(\frac{3}{5}\times\frac{1}{2}\right) \times \left(\frac{3}{5}\times\frac{1}{2}\right) \times \left(\frac{3}{5}\times\frac{1}{2}\right)$

$= \left(\frac{3}{5}\times\frac{1}{2}\right)^4$

$$x^a \times y^a = (x \times y)^a$$

5. (i) $2^4 \div 3^4 = \frac{2\times2\times2\times2}{3\times3\times3\times3} = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \left(\frac{2}{3}\right)^4$

(ii) $\left(\frac{3}{5}\right)^4 \div \left(\frac{1}{2}\right)^4 = \dfrac{\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}}{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}}$

$$= \left(\frac{\frac{3}{5}}{\frac{1}{2}}\right)\times\left(\frac{\frac{3}{5}}{\frac{1}{2}}\right)\times\left(\frac{\frac{3}{5}}{\frac{1}{2}}\right)\times\left(\frac{\frac{3}{5}}{\frac{1}{2}}\right) = \left(\frac{\frac{3}{5}}{\frac{1}{2}}\right)^4$$

$$x^a \div y^a = \left(\frac{x}{y}\right)^a$$

Ex.4 Simplify $\left[\left(\frac{2}{3}\right)^2\right]^3 \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times \frac{1}{6}$

Sol. $\left[\left(\frac{2}{3}\right)^2\right]^3 \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times \frac{1}{6}$

$= \left(\frac{2}{3}\right)^6 \times 3^4 \times \frac{1}{3} \times \frac{1}{6}$

$= \frac{2^6}{3^6} \times 3^4 \times \frac{1}{3} \times \frac{1}{6}$

$= 2^6 \times 3^{-6} \times 3^4 \times 3^{-1} \times 6^{-1}$

$= 2^6 \times 3^{-6} \times 3^4 \times 3^{-1} \times (2 \times 3)^{-1}$

$= 2^6 \times 3^{-6} \times 3^4 \times 3^{-1} \times 2^{-1} \times 3^{-1}$

$= 2^{6+(-1)} \times 3^{-1+4+(-1)+(-6)}$

$= 2^{6-1} \times 3^{-1+4-1-6}$

$= 2^5 \times 3^{-4} = \frac{2^5}{3^4} = \frac{32}{81}$

Ex.5 Find x so that $\left(\frac{2}{3}\right)^{-5} \times \left(\frac{2}{3}\right)^{-11} \times \left(\frac{2}{3}\right)^{8x}$

Sol. $\left(\frac{2}{3}\right)^{-5} \times \left(\frac{2}{3}\right)^{-11} \times \left(\frac{2}{3}\right)^{8x}$

$\left(\frac{2}{3}\right)^{(-5)+(-11)} = \left(\frac{2}{3}\right)^{8x}$

$\left(\frac{2}{3}\right)^{-5-11} = \left(\frac{2}{3}\right)^{8x}$

$\Rightarrow \left(\frac{2}{3}\right)^{-16} = \left(\frac{2}{3}\right)^{8x}$

$8x = -16 \qquad \therefore \quad x = -2$

So, If x is any rational number different from zero and a, b are any integers, then,

Law I: $x^a \times x^b = x^{a+b}$

Law II: $x^a \div x^b = x^{a-b}$

Law III: $(x^a)^b = x^{ab}$

Law IV: $x^a \times y^a = (x \times y)^a$

(where y is also a non-zero rational number)

Law V: $x^a \div y^a = \left(\frac{x}{y}\right)^a$

(where y is also a non-zero rational number)

Ex.6 Evaluate $\left(\frac{2}{3}\right)^4 \div \left(\frac{2}{3}\right)^4$

Sol. $\left(\frac{2}{3}\right)^4 \div \left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{4-4} \times \left(\frac{2}{3}\right)^0$

but $\left(\frac{2}{3}\right)^4 \div \left(\frac{2}{3}\right)^4 = \frac{\left(\frac{2}{3}\right)^4}{\left(\frac{2}{3}\right)^4} = 1$

the expression $= \left(\frac{2}{3}\right)^0 = 1$