GEOMETRIC OPTIC

BASIC CONCEPTS AND FORMULAE

1.1 Reflection : When a light ray incident on a smooth surface bounces back to the same medium, it is called **reflection.**

1.2 Laws of Reflection :

(i) Angle of incidence is equal to the anlge of reflection.

i.e., $i = r$

(ii) The incident ray, the reflected ray and the normal at the point of incidence, all lie in the same plane.

1.3 Spherical Mirror :

Spherical mirrors are of two types :

(i) Concave mirror : (ii) Convex mirror :

1.4 Mirror Formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

where u = distance of object from mirror ;

v = distance of image from mirror ;

and f = focal length of mirror.

1.5 Magnification Produced by Mirror :

$$\text{Magnification } M = \left(\frac{I}{O}\right) = -\frac{v}{u}$$

1.6 Refraction

Law of Refraction :

(i) The incident ray, the refracted ray and the normal to the surface separating the two media, all lie in the same plane.

(ii) Snell's Law : For two media, the ratio of sine of angle of incidence to the sine of the angle of refraction is constant for a beam of particular wavelength i.e.,

$$\frac{\sin i}{\sin r} = \text{constant} = \frac{n_2}{n_1} = {}_1n_2 \qquad ...(1)$$

1.7 Refractive index

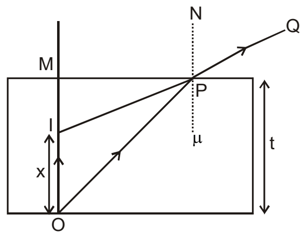
$$n = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}} = \frac{c}{v}$$

$$= \frac{\nu\lambda_{air}}{\nu\lambda_{medium}} = \frac{\lambda_{air}}{\lambda_{medium}} \qquad .....(4)$$

λ_{air} and λ_{medium} being wavelengths of light in air and medium respectively.

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{n_2}{n_1} = \left(\frac{c/v_2}{c/v_1}\right) = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \qquad ...(5)$$

1.8 Formation of image by refraction

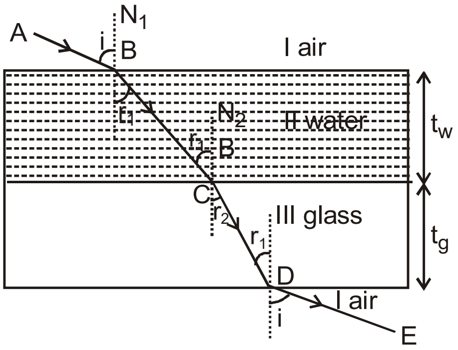


$$n = \frac{\text{Real depth(OM)}}{\text{Apparent depth (MI)}} = \frac{t}{t-x} \qquad ...(6)$$

where x is displacement or apparent shift.

$$\therefore \text{ The apparent shift, } x = \left(1 - \frac{1}{n}\right)t \qquad ...(7)$$

1.9 Refraction through a number of media :



$$ {}_{w}n_{g} = \frac{1}{{}_{a}n_{w} \times {}_{g}n_{a}} = \frac{{}_{a}n_{g}}{{}_{a}n_{w}} $$

2. CRITICAL ANGLE : TOTAL INTERNAL REFLECTION

The angle of incidence in denser medium for which the angle of refraction in rarer medium is 90° is called the **critical angle (C).**

If n_r and n_d are refractive indices for rarer and denser media, then

$$ \therefore \frac{\sin i}{\sin r} = \frac{n_2}{n_1} \text{ gives} $$

$$ \frac{\sin C}{\sin 90^\circ} = \frac{n_r}{n_d} = {}_{d}n_{r} $$

$$ \therefore \quad \sin C = {}_{d}n_{r} = \frac{1}{{}_{r}n_{d}} = \frac{1}{n} $$

2.1 Spherical Lenses :

There are two types of spherical lenses.

(i) Convex lens

(ii) Concave lens

Rules of Image Formation in Lenses

(i) The ray incident on lens parallel to the principal axis, after refraction through the lens pases thorugh the second focus (in convex lens) or appear to come from second focus in concave lens.

(ii) The ray incident on lens towards optical centre C, after refraction pass undeflected.

2.2 Thin Lens Formula

If u and v are object and image distances from a lens of focal length f, then thin lens formula is

$$ \frac{1}{f} = \frac{1}{v} - \frac{1}{u} $$

2.3 Magnification produced by a lens.

$$ m = \frac{I}{O} = \frac{v}{u} $$

2.4 Lens Maker's Formula

$$ \frac{1}{f} = ({}_{1}n_{2} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) $$

2.5 Power of a Lens

Power of lens, $P = \dfrac{1}{f(\text{in metres})}$ dipoters $= \dfrac{100}{f(\text{in cm})}$ diopters

2.6 Lens Immersed in a Liquid

$$\frac{1}{f_l} = ({}_l n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

n_l of liquid, n_g of lens

2.7 Thin Lenses in Contact

If two or more lenses of focal lengths f_1, f_2........ are placed in contact, then their equivalent focal length F is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + ... = \Sigma \frac{1}{f}$$

The power of combination

$$P = p_1 + p_2 + ... = \Sigma p.$$

2.8 Refraction Through a Prism

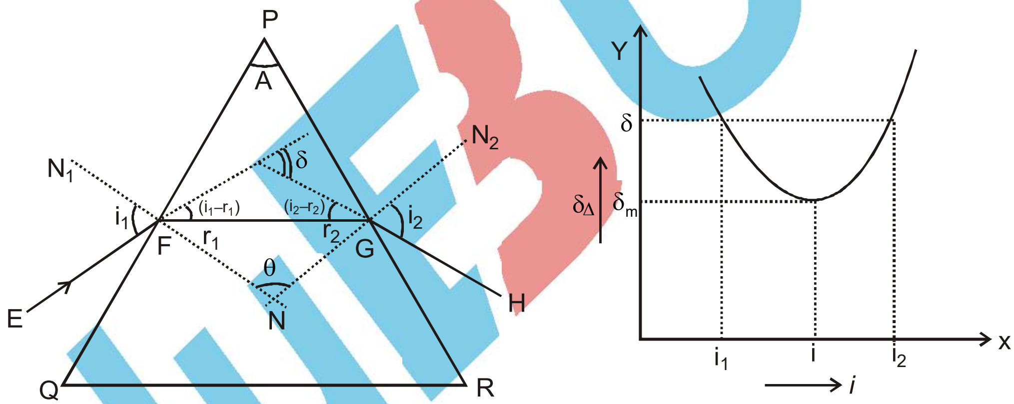
For a prism if A is the refracting angle of prism, then

$$r_1 + r_2 = A \qquad ...(1)$$

and

$$i_1 + i_2 = A + \delta \qquad ...(2)$$

$$n = \frac{\sin i_1}{\sin r_1} = \frac{\sin i_2}{\sin r_2} \qquad ...(3)$$



Minimum deviation : At minimum deviation the refracted ray within a prism is parallel to the base of prism. So

$i_1 = i_2 = i$(say)

$r_1 = r_2 = r$(say)

Then from equation (1) and (2),

$$r + r = A \text{ or } r = A/2 \qquad ...[(4)\ (a)]$$

$$i + i = A + \delta_m \quad \text{or} \quad i = \frac{A + \delta_m}{2} \qquad ...[(4)\ (b)]$$

∴ The refractive index of material of prism

$$n = \frac{\sin i}{\sin r} = \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}} \qquad ...(7)$$

Maximum deviation : For maximum deviation produced by a prism

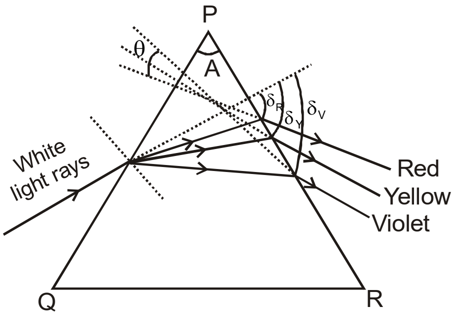
either i_1 or $i_2 = 90°$

,2.9 Disperison

The splitting of white light into its constituent colours is called the *dispersion.* When white light falls on a prism, it is broken into constituent colours within the prism. So the emergent light has a number of coloured beams, the violet being deviated most and red the least in visible region.

Thus the prism cause deviation as well as dispersion. If δ_v, δ_r and δ_y are the deviations caused by prism in violet, red and mean yellow rays, then for small angled prism.

Angular dispersion = $\delta_v - \delta_r = (n_v - n_r)A$



Dispersive power,

$$\omega = \frac{\text{Angular dispersion}}{\text{Mean deviation}} = \frac{\delta_v - \delta_r}{\delta_y} = \omega = \frac{\text{Angular dispersion}}{\text{Mean deviation}} = \frac{\delta_v - \delta_r}{\delta_y} = \frac{(n_v - n_r)}{(n_y - 1)} = \frac{(n_v - n_r)}{n_y - 1}$$

3. SCATTERING OF LIGHT

The light is scattered by air molecules. According to Lord Rayleigh the intensity of scattered light

$$I \propto \frac{1}{(\text{wavelength})^4} \Rightarrow I \propto \frac{1}{\lambda^4}$$

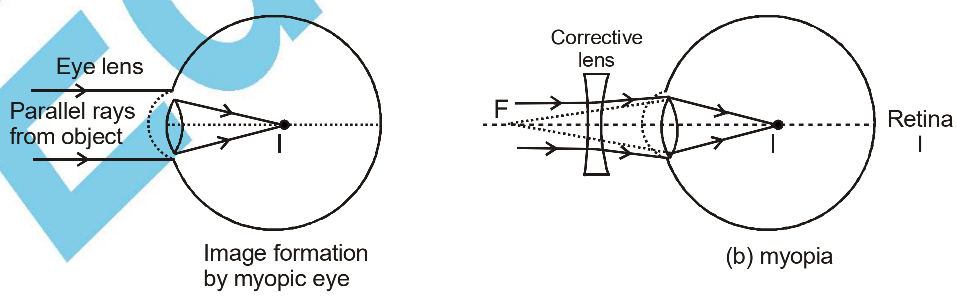
As $\lambda_{blue} < \lambda_{red}$, accordingly blue colour is scattered the most and red the least, so sky appears blue.

At the time of sunrise and sunset, blue colour is scattered the most and red colour enters our eyes, so sunrise and sunset appear red.

3.1 Defects of Eye and Their Correction

Due to growing age or otherwise, eye may suffere the following defects :

1. **Myopia or shortsightedness :** Myopia is the defect of eye in which a person can see only nearby objects, but fails to see the far away objects distinctly. This defect is due to



(a) decrease in focal length of the eye lens.

(b) Spreading of the eye-sphere.

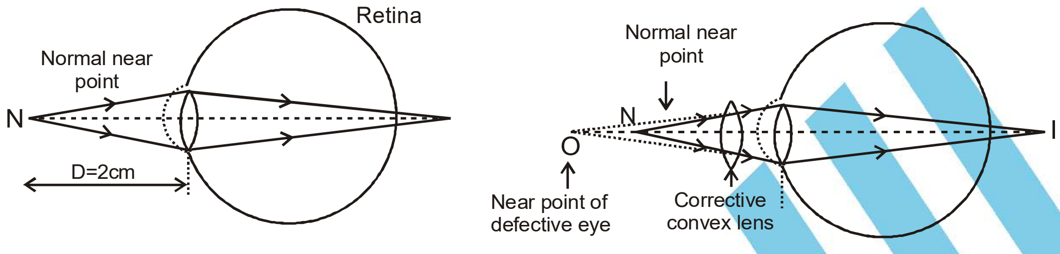
Due to these reason the image is formed in front of the retina.

Clearly, for elimination of myopia ***the focal length of corrective concave lens will be equal to the distance of far point of myopic eye from the eye lens.***

2. **Longsightedness of Hypermetropia :** Hypermetropia is the defect of eye in which a person can see only farther objects but fails to see nearer objects distinctly. this defect is due to

(a) Increase in focal length of eye lens. (b) Contraction of eye-sphere.

Due to these reasons the image of a nearby obect is formed behind the retina.



(a) Image formation by hypermetropic eye

(b) Corrected hypermetropia

3. **Presbyopia :** In growing age, the eye lens loses its flexibility of changing the focal length. Consequently, the near point of an eye is displaced farther and far point of the eye is displaced nearer, so that the eye is unable to see the nearby as well as far away objects. This defect of eye is called presbyopia.

This defect may be eliminated by using bifocal lenses.

4. **Astigmatism :** The defect of eye in which horizontal and vertical objects at the same distance are not focused at the retina clearly is called **astigmatism.** This arises when the cornea is not spherical in shape. For example cornea could have a larger curvature in vertical plane than in horizontal plane.

If astigmated eye sees a wire mesh or a shirt having horizontal and vertical lines, then verticl and horizontal lines are not equally well focused; if vertical lines are well focused, the horizontal lines may appear distorted or curved and vice versa. This defect may occur along with the myopia or hypermetropia.

Remedy : The astigmation is corrected by using a cylindrical lens having a cylindrical surface of a radius of curvature with an appropriately direction.

3.2 Optical Instruments (Microscopes and Telescopes)

(1) **Simple Microscope :** It consists of a convex lens of small focal length f.

If β = angle subtended by an image on eye

α = angle subtended by an object on eye, when object is at a distance of distinct vision

Magnifying power,

$$M = \frac{\beta}{\alpha} = \frac{D}{v}\left(1 + \frac{v}{f}\right)$$

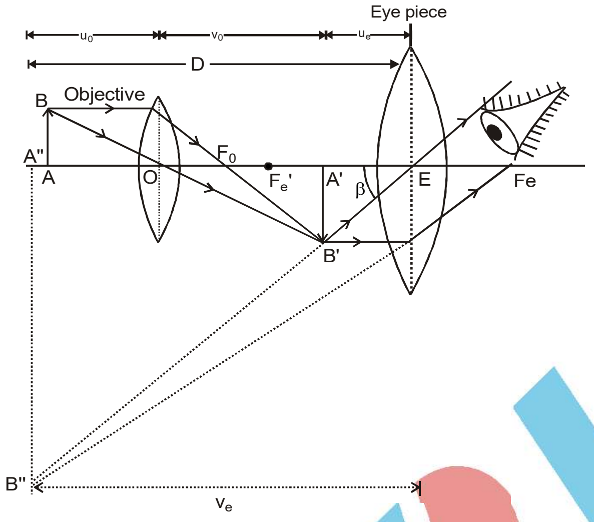
If the final image is at ∞, $v = \infty$ then $M = \frac{D}{f}$

If the final image is at a distance of distinct vision, $v = D$, $M = 1 + \frac{D}{f}$

(2) Compound Microscope :

Magnifying power of microscope,

$$M = \frac{\beta}{\alpha} (= m_0 \times m_e) = \frac{v_0}{u_0} \frac{D}{v_e} \left(1 + \frac{v_e}{f_e}\right)$$



The length of microscope,

L = length of tube

= sepration between lenses = $v_0 + u_0$

Special cases : (i) When final image is formed at a distance of distinct visio, v_e = D

$\therefore \quad M = -\frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$ and L = $v_0 + u_e$

In a compound microscope, object is placed, just beyond the focus i.e.,

$$M \approx -\frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$$

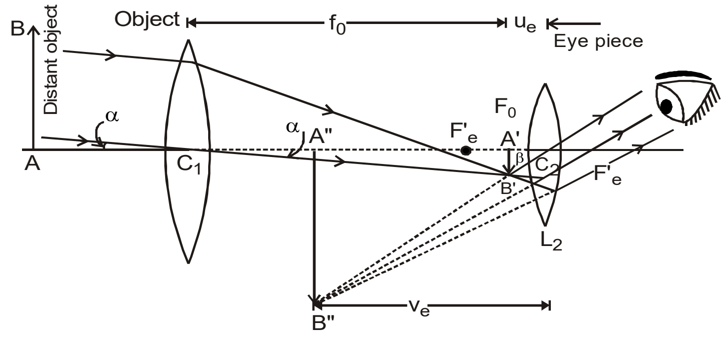
(ii) When final image is formed at infintiy, $v_e = \infty$, then

$$M = \frac{v_0}{u_0} \times \frac{D}{f_e} \quad \text{and } L = v_0 + f_e$$

$$M \approx -\frac{L}{f_0} \cdot \frac{D}{f_e}$$

(3) Astronomical Telescope (Refracting Telescope)

The magnifying power of telescope is



$$M = \frac{\text{Angle subtended by final image at eye}}{\text{Angle subtended by an object on eye}} = \frac{\beta}{\alpha}$$

$$= (m_0 \times m_e) = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{v_e}\right)$$

and Length of telescope $L = f_0 + u_e$

where v_e = distance of final image from eye lens

u_e = distance of real image A′ B′ from eye lens

Special cases (i) When final image is formed at a distance of distinct vision, then v_e = D

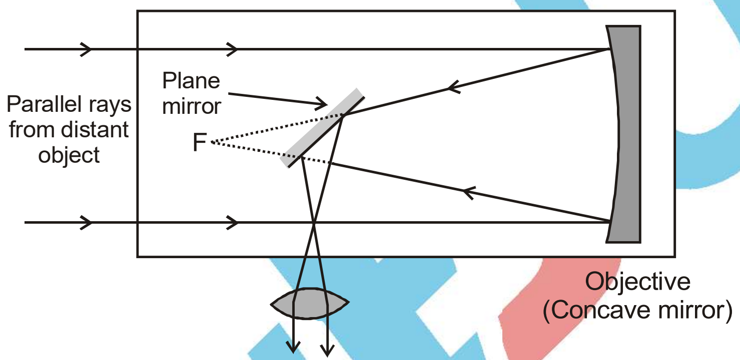
$$\therefore \quad M = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right) \quad \text{and } L = f_0 + u_e$$

(ii) When final image is formed at infinity, then $v_e = \infty$

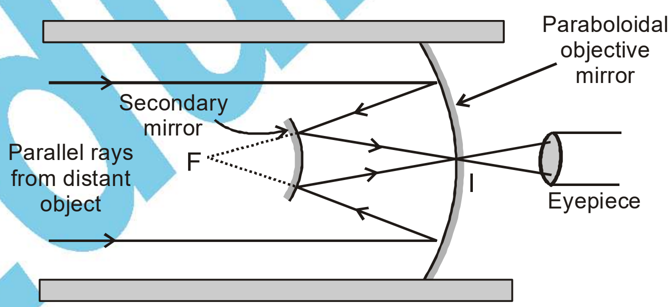
$$\therefore \quad M = -\frac{f_0}{f_e} \quad \text{and } L = f_0 + f_e$$

4. REFLECTING TELESCOPE

Newtonian reflecting telescope.



Newtonian reflecting telescope.



Cassegrain reflecting telescope.

For the final image formed at the least distnace of distinct vision,

$$m = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

For the final image formed at infinity,

$$m = \frac{f_0}{f_e} = \frac{R/2}{f_e}$$

WAVE OPTICS

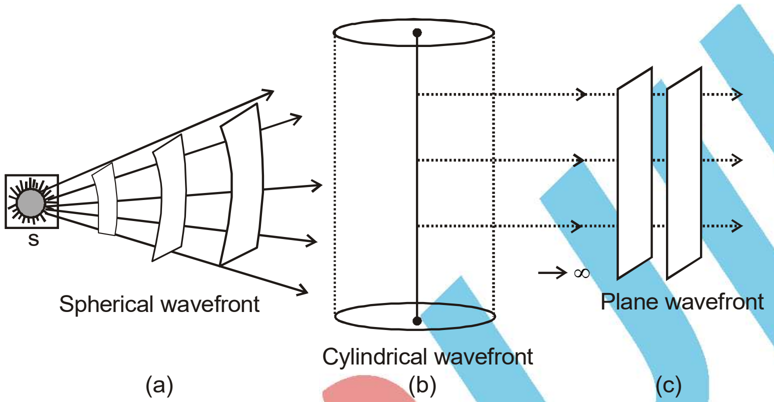
1 WAVE NATURE OF LIGHT : HUYGEN'S THEORY

1.1 Wavefront

A wavefront is defined as the continuous locus of all the particles which are vibrating in the same phase.

Types of Wavefronts

(i) Spherical Wavefront



(ii) Cylindrical Wavefront :

(iii) Plane Wavefront :

1.2 Coherent and Incoherent Sources of Light

The sources of light emitting wave having a constant initial phase difference are called coherent source.

The sources of light emitting wave with a random phase difference are called incoherent sources. For interference phenomenon, the source must be coherent.

1.3 Interference

Condtions of maxima and minima : If a_1 and a_2 are amplitudes of interfering waves and ϕ is the phase difference at a point under consideration, then

Resultant intensity at apoint in the region of superposition

$$I = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$$

$$= I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$$

Condition of maxima :

Phase difference, $\phi = 2n\pi$

or path difference, $\Delta = n\lambda$, n being integer

Maxima intensity, $I_{max} = A_{max}^2 = (a_1 + a_2)^2$

$$= a_1^2 + a_2^2 + 2a_1a_2$$

$$= I_1 + I_2 + 2\sqrt{I_1I_2}$$

Condition of minima : Phase difference, $\phi = (2n - 1)\pi$

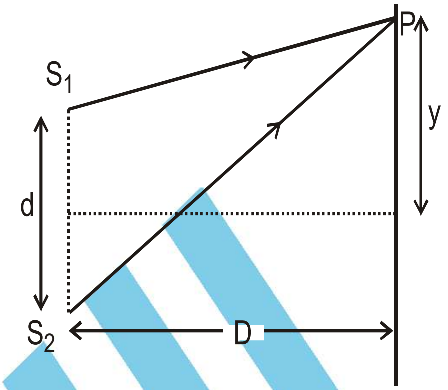
Path difference, $\Delta = (2n - 1)\dfrac{\lambda}{2}$, n = 1, 2, 3..........

Minimum intensity, $I_{min} = (a_1 - a_2)^2 = a_1^2 + a_2^2 - 2a_1a_2$

$$= I_1 + I_2 - 2\sqrt{I_1I_2}$$

1.4 Formulae of Young's Double Slit Experiment :

$$\Delta = \frac{y_n d}{D}$$



For maxima $\Delta = n\lambda$

$\therefore$ Position of nth maxima $y_n = \frac{nD\lambda}{d}$

$\therefore$ Position of nth minima $y_n = \left(n - \frac{1}{2}\right)\frac{D\lambda}{d}$

Fringe width : Fringe width is defined as the separation between two consectuive maxima or minima.

$$\beta = y_{n+1} - y_n = \frac{D\lambda}{d}$$

Angular fringe width, $\beta_\theta = \frac{\beta}{D} = \frac{\lambda}{d}$

Use of white light : When while light is used to illuminate the slit, we obtain an interference pattern consisting of a central white slit, we obtain an interference pattern consisting of a central white fringe having on both sides symmetrically a few coloured fringes and then uniform illumination.

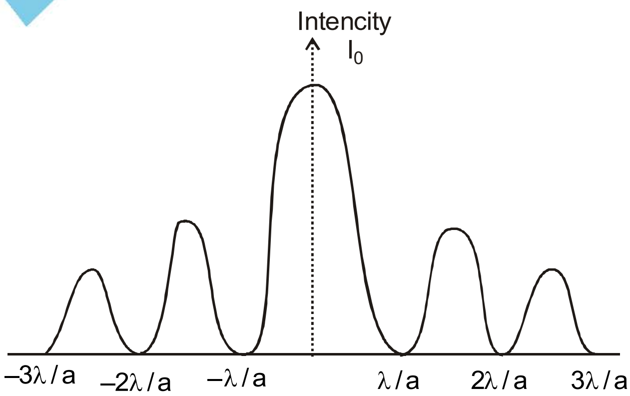
1.5 Diffraction due to a Single Slit

When a parallel beam of light is incident normally on a single slit, the beam is diffracted from the slit and the diffraction pattern consists of a very intense central maximum having on either sides minima, and secondary maxima alternately.

If a is width of slit and θ the angle of diffraction, then the directions of maxima

$$a\sin\theta = \left(n + \frac{1}{2}\right)\lambda$$

n = 1, 2, 3 ....



The positon of nth minima are given by

$$a \sin\theta = n\lambda$$

$\therefore$ Half-width of central maximum

$$\theta = \sin^{-1}\left(\frac{\lambda}{a}\right)$$

$\therefore$ Total width of central maximum,

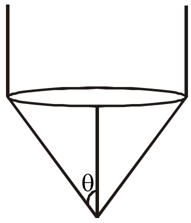
$$2\theta = 2\sin^{-1}\left(\frac{\lambda}{a}\right)$$

Linear Width : If D is the distance of screen from slit and y is the distance of nth minima from the centre of the principal maxima, then

Linear half-width of central maximum $y = \frac{\lambda D}{a}$

Total linear width of central maximum $2y = \frac{2\lambda D}{a}$

1.6 Resolving Power



1. Telescope : If a is the aperture of telescope and λ the wavelength, then resolving limit of telescope

$$d\theta \propto \frac{\lambda}{a}$$

For spherical aperture, $d\theta = \frac{1.22\lambda}{a}$

2. Microscope : For microscope, θ is the well resolved semi-angle of cone of light rays entering the telescope, then limit of resolution $= \frac{\lambda}{2n\sin\theta}$

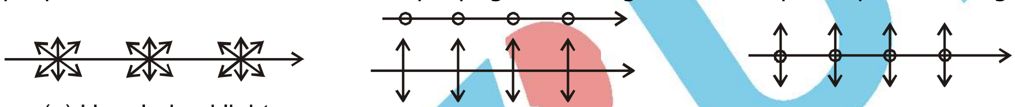
where n sin θ is called numerical aperture, NA

1.7 Polarisation

The phenomenon of asymmetry of vibrations about the direction of propagation of light is called polarisation.

1. Unpolarised Light : The light having vibrations of electric field vector in all possible directions perpendicular to the direction of wave propagation is called the oridinary (or unpolarised) light.

2. Plane Polarised Light : The light having vibrations of electric field vector in only one direction perpendicular to the direction of propagation of light is called plane polarised light.



(a) Unpolarised light (b) Polarised light (c) Partially polarized light

3. Polarisation by Reflection : ***Brester's Law :*** If unpolarised light falls on a transparent surface of refractive index n at a certain angle I_p, called polarising angle, then reflected light is plane polarised.

This is called Brewter's law. It is given by $n = \tan i_p$

Under this condition the reflected refracted rays are mutually pependicular i.e.,

$$i_p + r = 90°$$

where r is angle of refraction into the plane.

5. Malus Law : It states that if polarised light is passed through an analyser, the intensity of light transmitted $\propto \cos^2\theta$, where θ is angle between planes of transmission on polariser and analyser i.e.,

$$I = I_0 \cos^2\theta \qquad \text{(Malus Law)}$$

In incident light in unpolarised, then

$$I = \frac{I_0}{2},$$

since $(\cos^2\theta)_{average}$ for all directions 1/2.

6. Polaroid : Polaroid is a device to produce and detect plane polarized light.

Some Uses of Polaroids are :

(i) Sun glasses filled with polaroid sheets protect our eyes from glass.

(ii) Polaroids reduce heat light glass of motor cary being driven at light.

(iii) Polaroids are used in three-dimensional picture i.e., in holography.

7. Analysis of given light beam : For this given light beam is made incident on a polaroid (or Nicol) and the polaroid/Nicol is gradually rotated :

(i) If light beam shown no variabtion in intensity, then the given beam is umpolarised.

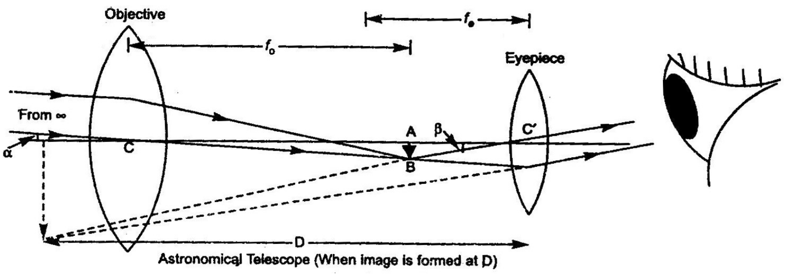
(ii) If light beam shows variation in intensity but the minimum intensity is non-zero, then the given beam is partially polarised.

(iii) If light beam shows variation in intensity and intensity becomes zero twice in a rotation then the given beam of light is plane polarised.

SOLVED PROBLEMS

1. ***Draw a ray diagram of an astronomical telescope in the the near point adjustment. Write down the epxression for its magnifying power.***

Sol.



Astronomical Telescope (When image is formed at D)

$$m = \frac{f_0}{f_e}$$

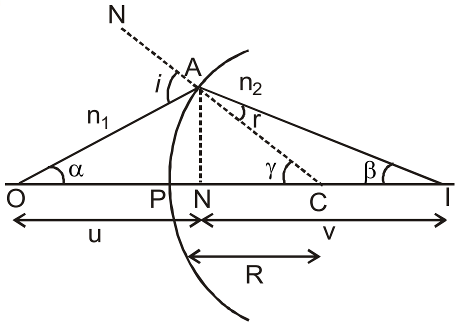
2. ***Draw a ray diagram to show the formation of the image of a point object placed in a medium of refractive index 'n_1' on the pricipal axis of a conved spherical surface of radius of curvature 'R' and refractive index 'n_2'. Using the diagram, derive the relation***

$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, ***where 'u' and 'v' have their ususal meanings.***

A converging lens of focal length 50 cm is placed coaxially in contact with an other lens of unknown focal length. If the combination behaves like a diverging lens of focal length 50 cm, find the power and nature of the second lens.

Sol. From ray diagram, i = α + γ

Since the aperture of the spherical refracting surface is small, i = tan α + tan γ



∴ $\tan\alpha = \frac{AN}{ON} \tan\gamma = \frac{AN}{NC}$

Also, ON ≈ OP & NC ≈ PC

[As the apeture is small]

∴ $\tan\alpha = \frac{AN}{OP} \tan\gamma = \frac{AN}{PC}$

∴ $i = \frac{AN}{OP} + \frac{AN}{PC}$ ...(1)

& γ = r + β [Exterior Angle]

∴ γ = r – β

r = tan γ – tan β [As the aperture is small]

∵ $\tan\gamma = \frac{AN}{NC} \approx \frac{AN}{PC}$ $\tan\beta = \frac{AN}{NI} \approx \frac{AN}{PI}$

$$r = \frac{AN}{PC} - \frac{AN}{PI} \quad ...(2)$$

But, $n = \frac{n_2}{n_1} = \frac{\sin i}{\sin r} \Rightarrow n_1 \sin i = n_2 \sin r$

For small aperture angles i & r must be very small, hence

$$n_1 i = n_2 r$$

$$n_1\left(\frac{AN}{OP}+\frac{AN}{PC}\right)=n_2\left(\frac{AN}{PC}-\frac{AN}{PI}\right) \Rightarrow \frac{n_2-n_1}{PC}=\frac{n_1}{OP}+\frac{n_2}{PI}$$

Using sign convention,

PC = + R

PI = +v

OP = – u

$$\frac{n_2-n_1}{R}=\frac{n_2}{v}-\frac{n_1}{u}$$

f_1 = +50 cm; f_2 = ?; f_{12} = – 50 cm.

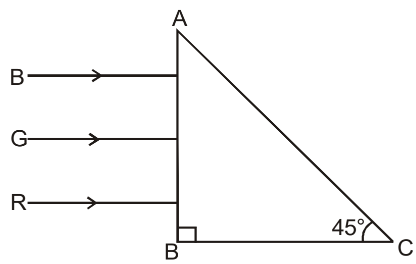
$$\frac{1}{f_{12}}=\frac{1}{f_1}+\frac{1}{f_2} \Rightarrow \frac{1}{-50}=\frac{1}{+50}+\frac{1}{f_2}$$

f_2 = – 25 cm = – 0.25 m

Power, $P_2=\frac{1}{f_2}=\frac{1}{-0.25}=-4D$

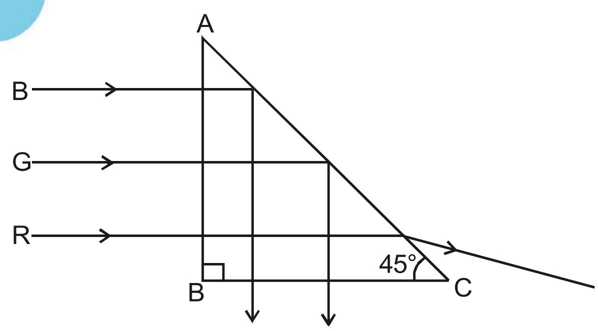
The second lens is concave type in nature.

3. ***Three rays of light – red (R), green (G) and blue (B) – are incident on the face AB of a right - angled prism ABC. The refractive indices of the mateial of the prism for red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. Trace the path of the rays through the prism. How will the situation change if these rays were incident normally on one of the facs of an equilaterial prism ?***



Sol. $\mu=\frac{1}{\sin C}=\frac{1}{\sin 45^\circ}=\sqrt{2}=1.414$

Hence, for refractive index greater than 1.414, the angle of incidence will be greater than the critical angle. Therefore, the green and blue rays will undergo total internal reflection on the face AC, whereas, the red light will suffere refraction.



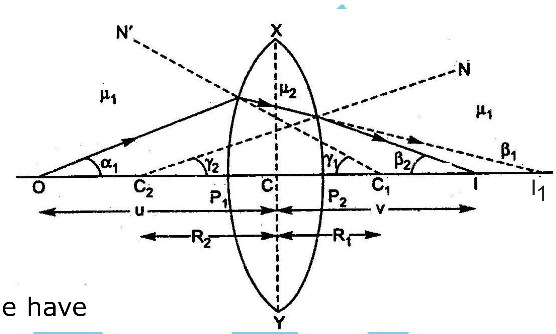
If these rays were incident normally on one of the faces of an equilateral prism, all the rays will suffere total internal reflection.

4. ***(a) Derive the relation $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ for a biconvex lens. How is the approximate value of the focal length of such a lens estimated in the laboratory ?***

(b) A beam of light converges to a point P. A lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (i) a convex lens of 20 cm focal length and (ii) a concave lens of 16 cm focal length ?

Sol. (a) Taking XP_1Y spherical surface into account, we have

$$\frac{\mu_2 - \mu_1}{P_1C_1} = \frac{\mu_2}{P_1I_1} + \frac{\mu_1}{P_1O}$$



Since P_1 lies close to C,

$$P_1O = CO,\ P_1I_1 \approx CI_1 \ \&\ P_1C_1 \approx CC_1$$

$$\frac{\mu_2 - \mu_1}{CC_1} = \frac{\mu_2}{CI_1} + \frac{\mu_1}{CO} \qquad \text{....(1)}$$

Taking XP_2Y spherical surface into account, we have

$$\frac{\mu_2 - \mu_1}{P_2C_2} = -\frac{\mu_2}{P_2I_1} + \frac{\mu_1}{P_2I}$$

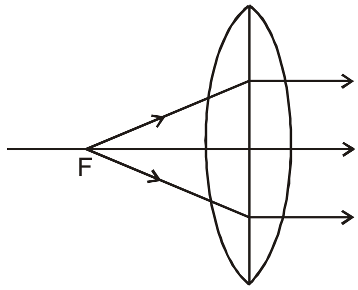
Since P_2 lies close to C,

$$P_2I = CI,\ P_2I_1 \approx CI_1 \ \&\ P_2C_1 \approx CC_1$$

$$\frac{\mu_2 - \mu_1}{CC_2} = -\frac{\mu_2}{CI_2} + \frac{\mu_1}{CI}$$

From (1) & (2),

$$\mu_1\left(\frac{1}{CO} + \frac{1}{CI}\right) = (\mu_2 - \mu_1)\left(\frac{1}{CC_1} + \frac{1}{CC_2}\right)$$



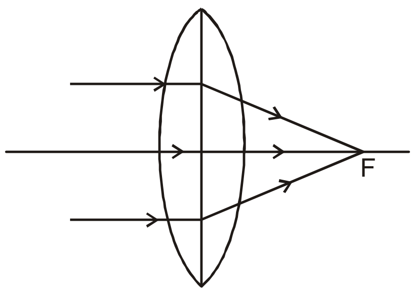
But, $CO = -u$; $CI = +u$; $CC_1 = +R_1$; $CC_2 = -R_2$

$$\mu_1\left(\frac{1}{-u} + \frac{1}{v}\right) = (\mu_2 - \mu_1)\left(\frac{1}{R_1} + \frac{1}{-R_2}\right)$$

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

First Principal Focus : is the position of the object on the pricniple axis, such that its image is formed at infinity.



$$\frac{1}{\infty} - \frac{1}{-f_1} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{...(4)}$$

Second Principle Focus : Is the position of the image on the principal axis of the lens, when object lies at infinity.

$$\frac{1}{f_2}-\frac{1}{\infty}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_2}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad ...(5)$$

From Equation (4) and (5), we find $f_1 = f_2 = f$

Hence, $$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad ...(6)$$

From (3) & (6), $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

It is known as thin lens formula.

Approximate value of focal length is estimated in the laboratory by obtaining a roughly sharp image of the surrounding object such as tree or building.

(b) (i) u = + 12 cm; f =+20 cm

Hence, $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

$$\frac{1}{v}=\frac{1}{u}+\frac{1}{f}=\frac{1}{12}+\frac{1}{20}=\frac{2}{15}$$

u = +7.5 cm

(ii) u = +12 cm; f = − 16 cm

Hence, $\frac{1}{v}=\frac{1}{u}+\frac{1}{f}=\frac{1}{+12}+\frac{1}{-16}=\frac{1}{+48}$

v = + 48 cm

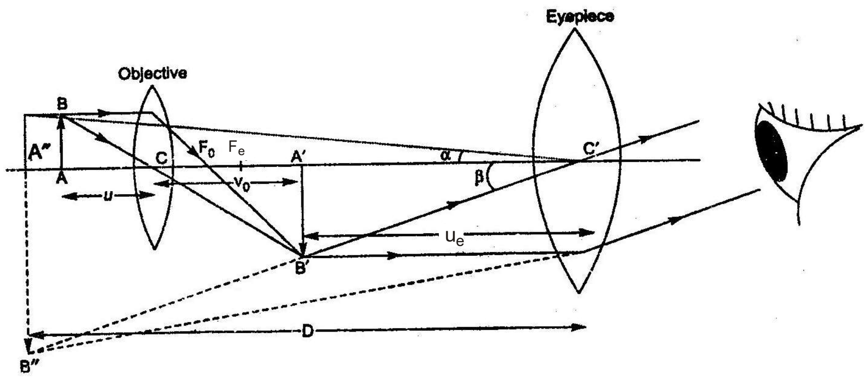
5. ***(a) Draw a labelled ray diagram to show the formation of an image by a compound microscope. Write the expression for its magnifying power.***

(b) How does the resolving power of a compound microscopechange, when

(i) refractive index of the medium between the object and the objective lens increases; and (ii) wavelength of the radiation used is increased ?

Sol. Magnification, (a) $m=-\frac{L}{f_0}\left(1+\frac{D}{f_e}\right)$ [for final image at D]

(b) $m=-\frac{L}{f_0}\times\frac{D}{f_e}$ [for final image at ∞]



(b) The resolving power, $\frac{1}{d}=\frac{2\mu\sin\theta}{1.22\lambda}$

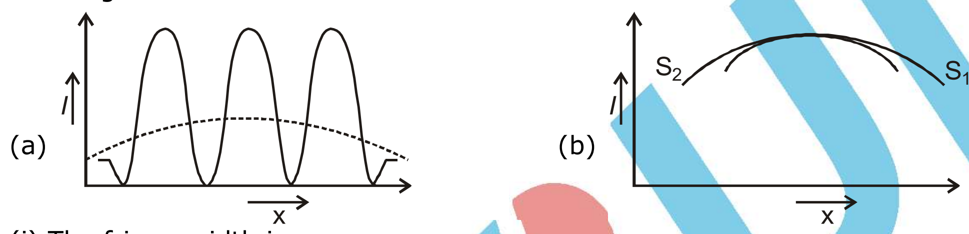
(i) Resolving power increases with the increase in refractive index

(ii) Resolving power decreases with the increase in wavelength of the radiation

6. ***What is interference of light ? Write two essential conditions for sustained interference pattern to be produced on the screen.***
Draw a graph showing the variation of intensity versus the position on the acreen in Young's experiment when (a) both the slits are opened and (b) one of the slits is closed.
What is the effect on the interference pattern in Young's Double slit experiment when :
(i) Screen is moved closer to the plane of slits ?
(ii) Separation between the two slits is increased.
Explain your answer in each use.

Sol. The phenomenon of redistribution of energy in the medium due to superposition of two light waves is called interference of light.

Condition for Sustained Interference :
- The two sources of light should emit light continuous.
- The light waves should be Monochromatic.



(i) The fringe-width increases
(ii) The fringe-width decreases

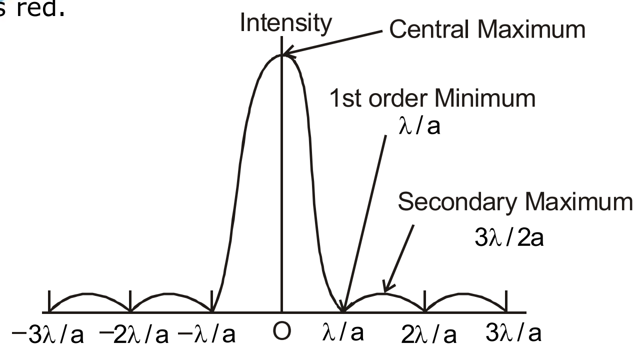
Fringe-width, $\beta = \frac{\lambda D}{d}$

$\beta \propto D$, D $\rightarrow$ distance between screen and slits.

$\beta \propto \frac{1}{d}$, d $\rightarrow$ distance between slits.

7. ***What is diffraction of light ? Draw a graph showing the variation of intensity with angle in a single slit diffraction experiement. Write one feature which distinguishes the observed pattern from the single slit pattern from the double slit interference pattern.***
How would the diffraction pattern of a single slit be affected when :
(i) the width of the slit is decreased
(ii) the monochromatic source of light is replaced by a source of white light ?

Sol. The phenomenon of bending of light around the sharp corners and spreading into the regions of geometrical show is called diffraction.
Interference (Single slit pattern) : Produced due to two different wavefronts.
Diffraction (Double slit pattern) : Produced due to different parts of same wavefronts.
(i) Fringe-width increases
(ii) Coloured fringes will be observed. Central fringe is white, fringe closest to it is blue and the farthest is red.



8. ***(a) What are coherent source of light ? State two conditions for two light sources t be coherent.***

(b) Derive a mathematical expression for the width of interference fringes obtained in Young's double slit experiment with the help of a suitable diagram.

(c) Give relation between deviation angle and refractive index.

Sol. **(a) Coherent Sources :** When two sources are produced from a single source, the amplitude, wave length of the source are same and they are in constant phase difference, then such sources are know as coherent sources.

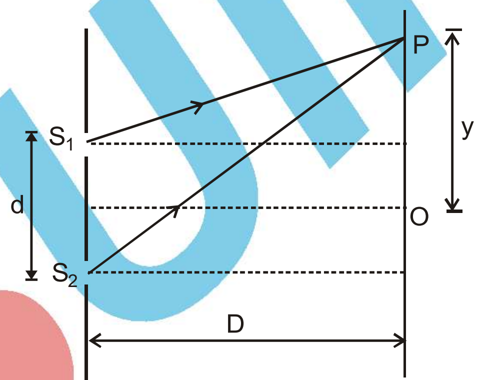
Condition for Sustained Interference :

- The two sources of light should emit light continuous.
- The light wave should be Monochromatic.

(b) Derivation :

The path difference between the light waves from slits S_1 & S_2

$$x = S_2P - S_1P$$



But $(S_1P)^2 = D^2 + \left(y - \frac{d}{2}\right)^2$

$(S_2P)^2 = D^2 + \left(y + \frac{d}{2}\right)^2$

& ∴ $(S_2P)^2 - (S_1P)^2 = \left(y + \frac{d}{2}\right)^2 - \left(y - \frac{d}{2}\right)^2$

$\Rightarrow (S_2P - S_1P)(S_2P + S_1P) = 4y.\frac{d}{2} = 2yd$

$\Rightarrow x.(S_2P + S_1P) = 2yd$

$$x = \frac{2yd}{(S_2P + S_1P)}$$

But, $S_2P \approx S_1P \approx D$

$$x = \frac{2yd}{2D} = \frac{yd}{D}$$

$$x = \frac{yd}{D}$$

For Bright fringes, $\frac{yd}{D} = n\lambda$

Hence, $y = \frac{n\lambda D}{d}$, whre n = 0, 1, 2, ...

For dark fringes $\frac{y'd}{D} = \frac{2n+1}{2}\lambda$

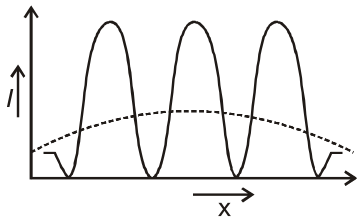
$y' = \frac{(2n+1)\lambda D}{2d}$, where n = 0, 1, 2, ...

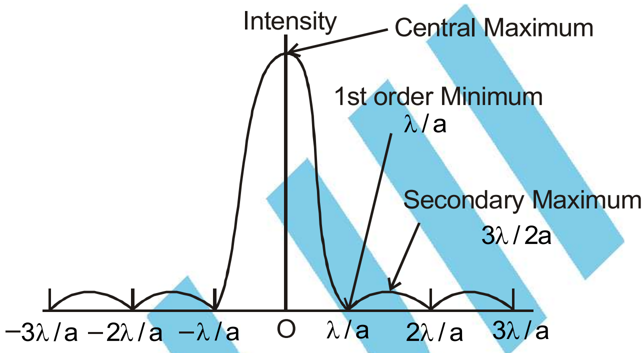
Fringe Width, $\beta = y_n - y_{n-1} = \frac{n\lambda D}{d} - \frac{(n-1)\lambda D}{d} = \frac{\lambda D}{d}$ $\quad \therefore \beta = \frac{\lambda D}{d}$

For dark fringes, $\beta = y'_n - y'_{n-1} = \frac{(2n+1)\lambda D}{2d} - \frac{(2n-1)\lambda D}{2d} = \frac{\lambda D}{d}$ $\quad \therefore \beta = \frac{\lambda D}{d}$

(c) $\delta = (n - 1)A$

9. ***Two narrow slits are illuminated by a single monochromatic source. Name the pattern obtained on the screen. One of the slits is now completely covered. What is the name of the pattern now obtained on the screen ? Drawn intensity pattern obtained in the two cases. Also write two differences between the patterns obtained in the above two cases.***





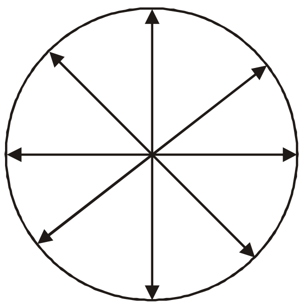
Interference	Diffraction
Produced due to two different wavefronts Fringe width is of same size. Fringes have same intensity	Produced due to different parts of same wavefronts Central fringe is twice as wide as other fringes Intensity of fringes decreases as we go to successive maxima away from the centre

10. ***(a) How is wavefront different from a ray ? Draw the geometrical shape of the wavefronts when (i) light diverges from a point source, and (ii) light emerges out of a convex lens when a point source is placed at its focus.***

(b) State Huygen's principle. With the help suitable diagram, prove Snell's law of refraction using Huygen's principle.

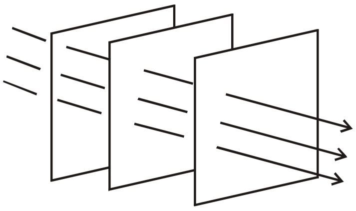
Sol. (a) Wavefront acts as a source and rays from each point on the wavefront moves in all direction, whereas the ray moves in a particular direction.

(i) When light diverges from a point source



Spherical Wavefront

(ii) When light emerges out of a convex lens when a point source is placed at its focus



Plane Wavefront

(b) Huygens' Principle

(i) Every point on a primary wavefront acts as a source of secondary wavelets. The secondary wavelet sends out disturbances in all directions just as the primary source of light.

(ii) The new position of the secondary wavefronts is the envelope of the primary wavefront.

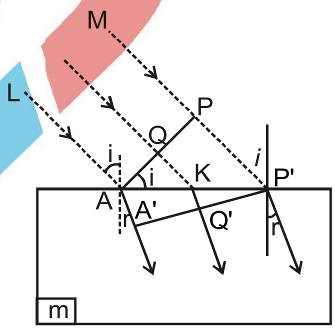
Wave Theory : Laws of Refraction

Consider a ray LM, whose wavefront AP after time t, then

$$\frac{AA'}{v} = \frac{PP'}{c}$$

where v and c are the speed of light in medium of refractive index M and in air. Time taken by light to go from Q to Q′

$$t = \frac{QK}{c} + \frac{KQ'}{v} \qquad ...(1)$$



In right angled ΔAQK

KQ′ = KP′ sin r

$$t = \frac{AK\sin i}{c} + \frac{KP'\sin r}{v}$$

$$= \frac{AK\sin i}{c} + \frac{(AP'-AK)\sin r}{v}$$

$$= \frac{AP'\sin r}{v} + AK\left(\frac{\sin r}{c} - \frac{\sin r}{v}\right)$$

For rays from different points on the incident wavefront, the values of AK are different. The rays from different points on the AP wavefront will the same time to reach the wavefront A′ P′. Hence equations (2) must be independent of AK. i.e.,

$$\frac{\sin i}{c} - \frac{\sin r}{v} = 0 \quad \Rightarrow \quad \frac{c}{v} = \frac{\sin i}{\sin r}$$

But $\frac{c}{v} = \mu$

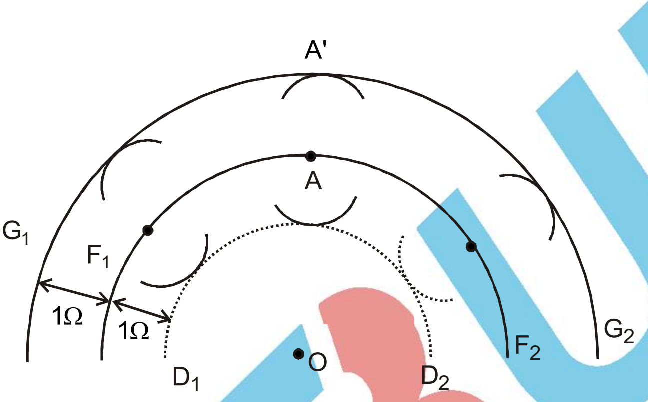
$$\therefore \quad \frac{\sin i}{\sin r} = \frac{c}{v} = \mu$$

Thus, when a ray of light under goes refraction, the ratio of sine of angle of incidence to the angle of refraction is a constant for a given pair of media and is equal to the refractive index for the given pair of media.

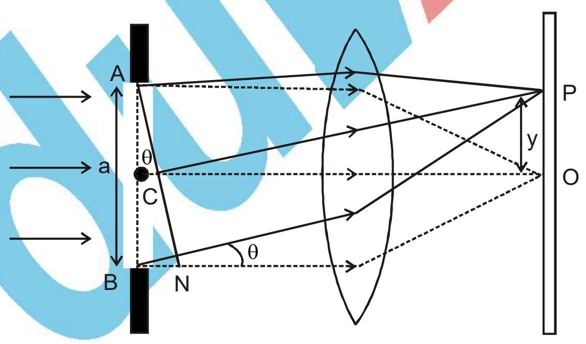
11. ***Using Huygens' principle, draw a diagram to show propagation of a wave-front originating from a monochromatic point source.***

Describe diffraction of light due to a single slit. Explain formation of a pattern of fringes obtained on the screen and plot showing variation of intensity with angle θ in single slit diffraction.

Sol. **1.** Every point on primary wavefront acts as a source of secondary wavelets. The secondary wavelet sends out disturbances in all directions just as the primary source of light.



2. The new position of the secondary wavefronts is the envelope of the primary wavefront.



From figure, BN = AB sin θ = a sin θ

Choosing, BN = λ and $\theta = \theta_1$, we have

$\lambda = a \sin \theta_1$

$\therefore \sin \theta_1 = \frac{\lambda}{a}$, where θ_1 is the angle up to which the central maximum extends on either side.

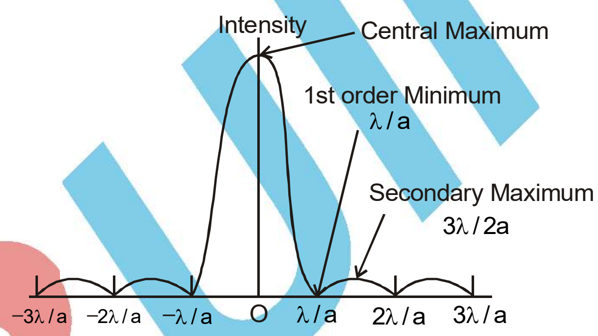
Such an angular position on the screen will represent the first secondary minimum. To understand it, we assume the slit to be divided into two equal halves, the wavelets from the corresponding points of the two halves of the slit will have a path difference of $\frac{\lambda}{2}$, i.e., the wavelets from each half will reach point P in opposite phase. Hence, for second secondary minimum,

$$2\lambda = a \sin \theta_2$$

$$\sin \theta_2 = \frac{2\lambda}{a}$$

& $\sin\theta_n = \frac{n\lambda}{a}$ for n^{th} secondary minima

If y_n is the distance of n^{th} secondary minimum from the center of the secreen then



$$\tan\theta_n = \frac{OP}{CO} = \frac{y_n}{D}$$

For small value of θ_n,

$$\sin \theta_n = \tan \theta_n$$

$$\frac{y_n}{D} = \frac{n\lambda}{a} \Rightarrow y_n = \frac{n\lambda D}{a}$$

$$\therefore \quad \beta = y_n - y_{n-1} = \frac{\lambda D}{a}$$

For first secondary maxima

$\sin \theta'_1 = \frac{3\lambda}{2a}$; = [Since, the wavelets from each half will reach point P such that out of three equal parts two will cancel out leaving one part of wavelet to produce the Bright fringes.]

Similarly, $\sin\theta_2 = \frac{5\lambda}{2a}$

& $\sin\theta_n = \frac{(2n+1)\lambda}{2a}$ for n^{th} secondary maxima

$$\therefore \quad \beta' = y'_n - y'_{n-1} = \frac{\lambda D}{a}$$

Both β & β' are independent of value of n. Hence, all the secondary maxima & minima are of the same width.

1. Calculate time taken by light to travel 1 cm thickness of glass of $\mu = 1.5$.

2. When a wave undergoes reflection at a denser medium, what happens to its phase ?

EXERCISE - I UNSOLVED PROBLEMS

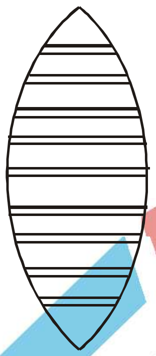
3. When a wave undergoes refraction at a denser medium, what happens to its phase ?

4. Which of the two colours yellow and violet travels slower in glass prism ?

5. What is the value of refractive index of a medium of polarizing angle 60° ?

6. What type of lens is an air bubble Inside water ?

7. A lens shown in figure is made of two different materials. A point object is placed on the pricipal axis of the lens. How many images will be obtained ?



8. A person looking at a mesh of crossed wires is able to see the vertical wires more distinctly than the horizotnal wires. What is the defect due to ? How is such a defect of vision corrected ?

9. Using the data given below, state which two of the given lenses will you prefere to construct a best possible (i) telescope (ii) microscope. Also, indicate which of the selected lenses is to be used as an objective and as an eye piece in each case.

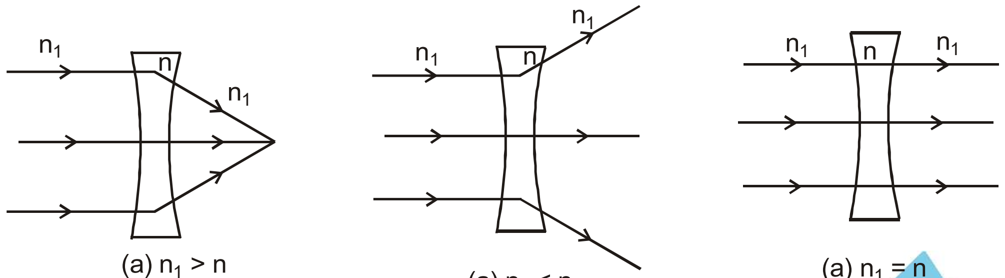
Lenses	Power (P)	Apeture(A)
L_1	6D	1cm
L_2	3D	8cm
L_3	10D	1cm

10. A compound microscope has a magnification of 30. The focal length of the eye piece is 5 cm. Assuming that final image is formed at the least distance of distinct vision (25 cm), calculate the magnification produced by the objective.

11. The radius of curvature of each surface of a convex lens of refractive index 1.5 is 40 cm. Calculate its power.

12. Why there is no dispersion of light refracted through a rectangular glass slab ?

13. The refractive index of the material of a concave lens is n. It is immersed in a medium of refractive index n_1. A parallel beam of light is incident on the lens. Trace the path of emergent rays in each of the following cases :



(a) $n_1 > n$ (a) $n_1 < n$ (a) $n_1 = n$

14. Which phenomena establish the wave nature of light ?

15. No interference pattern is detected when two coherent sources are infinitely close to each othe. Why ?

16. The widths of two slits in Young's experiment are in the ratio 9 : 4. Calculate the intensity ratio in the interference pattern.

17. Name the type of wave front that corresponds to a beam of light.
(i) coming from a convex lens when point source is placed at its focus,
(ii) coming from a very far off source,
(iii) diverging radially from a point source.

18. In young's experiment, the width of the fringes obtained with the light of wavelength 6000Å is 2.00 mm. What will be the fringe width of the entire apparatus is immersed in a liquid of μ = 4/3.

19. In Young's double slit experiment, the angular width of a fringe formed on a distant screen is 1°. The wavelength of light used is 6000Å. What is the spacing between the slits ?

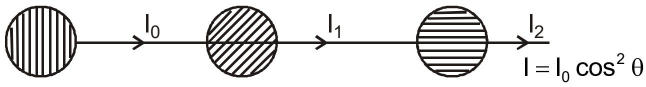
20. Let us list some of the factors which could possibly influence the speed of wave
(i) Nature of source
(ii) Direction of propagation
(iii) Motion of source and /or observer
(iv) Wavelength
(v) Intensity of wave.
On which of these factors, if any, does
(a) The speed of light in vacuum
(b) Speed of light in a medium (say glass or water) depend ?**1.** Two thin lenses of power +6D and –2D are in contact. What is the focal length of the combination.

2. (i) State the principle on which the working of an optical fibre is based.
(ii) What are the necessary condition for this phenomenon to occur.

EXERCISE - II BOARD PROBLEMS

GEOMETRIC OPTICS

3. Define the term 'Linearly polarised light' why does the intensity of transmitted light become maximum, when a polaroid sheet is rotated between two crossed polaroid



4. **(a)** (i) Draw a labelled ray diagram to show the formation of image in an astronomical telescope for a distant object.

(ii) Write three distinct advantage of a reflecting type telescope over a refracting type telescope.

(b) A convex lens of focal length 10 cm is placed coaxially 5 cm away from a concave lens of focal length 10 cm. If an object is placed 30 cm in front of the convex lens, find the position of final image formed by the system.

5. **(a)** With the help of a suitable ray diagram, derive mirror formula for a concave mirror.

(b) The near point of a hypermetropic person is 50 cm from the eye. What is the power of the lens required to enable. What is the power of the lens required to enable the person to read clearly a book held at 25 cm from the eye ?

6. When light travels from a rarer to denser medium, the speed decreases. Does this decreases in speed imply a decrease in the energy carried by the light wave ? justify your answer.

7. A converging lens is kept coaxially in contact with diverging lens both the lenses being of equal focal length. What is the focal length of the combination.

8. A convex lens is used to obtain a magnified image of an object on a screen 10m from the lens. If the magnification is 19, find the focal length of the lens.

9. How does an unpolarised light get polarised when passed thorugh a polaroid ?

Two polaroids are set in crossed position. A third polaroid is placed between the two making angle θ wtih the pass axis of the first polaroid. Write the expression for the intensity of light transmitted from the second polaroid. In what orientations will the transmitted intensity be

(i) minimum & (ii) maximum

10. (i) Draw a neat labelled ray diagram of an astronomial telescope in normal adjustment. Explain briefly its working.

(ii) An astronomical telescope uses two lenses of powess 10 D and 1D. What is its magnifying power in normal adjustment ?

11. (i) Draw a neat labelled ray diagram of a compound microscope. Explain its working ?

(ii) Why must both the objective and the eye-piece of a compound microscope have short focal length ?

12. How would the angular separation of interference fringes in Young's double slit experiment change when the distance between the slits and screen is doubled ?

13. In young's double slit experiment, monochromatic light of wavelength 630 nm illuminates the pair of slits and produces an interference pattern in which two consecutive bright fringes are separated by 8.1 mm. Another source of monochromatic light produces the interference pattern in which the two consecutive bright fringes are separated by 7.2 mm. Find the wavelength of light from the second source.

What is the effect on this interfernece fringes if the monochromatic source is replaced by a source of white light

14. **(a)** In a single slit diffraction experiment a slit of width 'd' is illuminated by red light of wavelenth 650 nm. For what value of 'd' will

(i) the first minimum fall at an angle of 30°, and

(ii) the first maximum fall at an angle of diffraction of 30° ?

(b) Why does the intensity of the secondary maximum become less as compared to the central maximum ?

15. In young's double slit experiment, the two slits 0.12 mm apart are illuminated by monochromatic light of wavelength 420 nm. The screen is 1.0 m away from the slits.

(a) Find the distance of the second

(i) bright fringe,

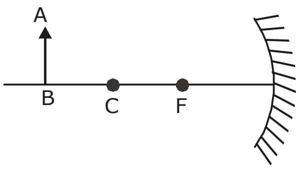
(ii) dark fringe from the central maximum

(b) How will the fringe pattern change if the screen is moved away from the slits ?

16. How does the angular separation between fringes in single-slit diffraction experiment change when the distance of separation between the slit and screen is doubled ?

17. For the same value of angle of incidence, the angles of refraction in three media A, B and C are 15°, 25° and 35° respectively. In which mdium would the velocity of light be minimum ?

18. An object AB is kept in front of a concave mirror as shown in the figure.

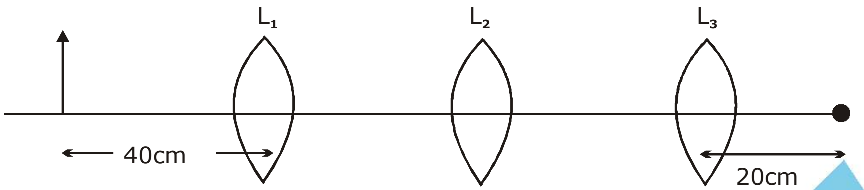


(i) Complete the ray diagram showing the image formation of the object.

(ii) How will the position and intensity of the image be affected if the lower half of the mirror's reflecting surface is painted black ?

19. Draw a labelled ray diagram of a relfecting telescop. Mension its two advantages over the refracting telescoe.

20. You are given three lenses L_1, L_2 and L_3 each of focal length 20 cm. An object is kept at 40 cm in front of L_1, as shown. The final real image is formed at the focus 'I' of L_3. Find the separations between L_1, L_2 and L_3.



21. (a) In Young's double slit experiment, derive the condition for (i) constructive interference and (ii) destructive interference at a point on the screen.

(b) A beam of light consisting of two wavelengths, 800 nm and 600 nm is used to obtain the interference fringes in a Young's double slit experiment on a screen placed 1.4 m away. If the two slits are separated by 0.28 mm, calculate the least distance from the central bright maximum where the bright fringes of the two wavelengths coincide.

22. (a) How does an unpolarized light incident on a polaroid get polarized ?

Describe briefly, with the help of a necessarty diagram, the polarization of light by reflection from a transparent medium.

(b) Two polaroids 'A' and 'B' are kept in crossed position. How should a third polaroid 'C' be placed between them so that the intensity of polarized light transmitted by polaroid B reduces to $1/8^{th}$ of the intensity of unpolarized light incident on A ?

23. A parallel beam of light of 600nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1.2 m away. It is observed that the first minimum is at a distance of 3 mm from the centre of the screen. Calculate the width of the slit.

24. A convex lens of focal length f_1 is kept in contact with a concave lens of focal length f_2. Find the focal length of the combination.

25. (a) What is linearly polarized light ? Describe briefly using a diagram how sunlight is polarised.

(b) Unpolarised light is incident on a polaroid. How would the intensity of transmitted light change when the polaroid is rotated ?

ANSWER KEY

EXERCISE - I UNSOLVED PROBLEMS

1. 0.5×10^{-10} s

2. Reflection of waves of a denser medium causes a phase change of 180°.

3. No phase change.

4. Violet colour travels slower. This is because $\mu_v > \mu_r$ and $v \propto 1/\mu$

5. $\sqrt{3}$ **6.** Concave lens

7. As the lens is made of two different materials, it has two refractive indices and hence, two distinct images will be formed.

8. This defect is astingmatism, which is caused due to improper curvature of retina. It can be corrected by the use of cylindrical lens.

9. For a telescope, lens L_2 is chosen as objective, s its aperture is largest. The lens L_3 is choen as eye piece as its focal length is smaller. For a microscope, lens L_3 is chosen as objective because its focal length is smaller. Lens serves as eye piece because its focal length not largest. This is required to secure greater magnifying power of the microscope.

10. 5 **11.** 2.5 D **14.** Interference, diffraction and polarization of light.

15. As fringe width, $\beta = \frac{\lambda D}{d}$

When d is very small, the fringe width will be too large and occupy the whole screen. Hence the pattern will not be seen.

16. 25 : 1 **17.** (i) Plane, (ii) Plane, (iii) Spherical **18.** 1.5 mm **19.** 0.0344 mm

20. (a) Speed of light in vacuum is constant. It does not depend upon any factor.

(b) The speed of light in a medium like water or glass

(i) Does not depend upon the nature of the source,

(ii) Does not depend upon the direction of propgation, when the medium is isotropic.

(iii) Does not depend upon the motion of the source w.r.t. the medium, but depends on motion of the observer relative to the medium.

(iv) Depends on wavelength of light, being lesser for shorter wavelength and vice-versa.

(v) Does not depend upon intensity of light.

EXERCISE - II BOARD PROBLEMS

1. $f = 25$ cm **3.** $\theta = 45°$ **4.** $v' = 15$cm, $v = \infty$ **5.** $f = 50$ cm, $P = 2D$

7. as $f_1 = -f_2$ $\therefore$ $f = \infty$

8. $v = -19u$, $f = 0.5$ m

9. $I = (I_0\cos^2\theta)\cos^2\theta(90° - \theta) = I_0(\cos\theta \sin\theta)^2 = I_0 \sin^2 2\theta/4$, max $\theta = 45°$, min $\theta = 0°$

10. $m = \frac{f_0}{f_e} = \frac{P_e}{P_0} = \frac{10}{1} = 10$

12. Will not change

13. $\beta = \frac{\lambda D}{d}, \frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2}, \lambda_2 = 560\,\text{nm}$

14. **(a)** (i) $d\sin\theta = n\lambda$, $d = 13 \times 10^{-7}$ m (ii) $d\sin\theta = (2n+1)\frac{\lambda}{2}$, $d = 1950$ nm

15. $x_2 = \frac{2\lambda D}{d} = 7$ mm [maxima] ; $y_2 = \frac{(2n-1)\lambda D}{2d} = 5.25$ (minima)

16. No change **17.** Medium

20. Dist. between L_1 & L_2 = 60 cm L_2 & L_3 can have are value **20.** $X_n = 12 \times 10^{-3}$ mm