Properties of Multiplication of Rational Numbers

Closure Property

Definition:

If we multiply any two rational numbers, the result is always a rational number.

That means:

If a and b are rational numbers, then a × b is also a rational number.

Example 1:

$(\frac{2}{5}) \times (\frac{3}{4}) = \frac{6}{20} = \frac{3}{10} \rightarrow$ a rational number

So, closure property holds.

Example 2:

$(-\frac{7}{8}) \times (\frac{2}{3}) = -\frac{14}{24} = -\frac{7}{12} \rightarrow$ a rational number

Closure property is true.

Commutative Property

Definition:

Changing the order of numbers does not change the result in multiplication.

That means:

a × b = b × a

Example 1:

$(\frac{2}{3}) \times (\frac{5}{7}) = \frac{10}{21}$

$(\frac{5}{7}) \times (\frac{2}{3}) = \frac{10}{21}$

Both are equal.

Example 2:

$(-\frac{4}{9}) \times (\frac{1}{2}) = -\frac{4}{18} = -\frac{2}{9}$

$(\frac{1}{2}) \times (-\frac{4}{9}) = -\frac{4}{18} = -\frac{2}{9}$

Commutative property holds.

Associative Property

Definition:

When multiplying three or more rational numbers, the way they are grouped does not change the result.

That means:

(a × b) × c = a × (b × c)

Example 1:

Let a = $\frac{1}{2}$, b = $\frac{2}{3}$, c = $\frac{3}{4}$

$(\frac{1}{2} \times \frac{2}{3}) \times \frac{3}{4} = (\frac{1}{3}) \times \frac{3}{4} = \frac{3}{12} = \frac{1}{4}$

$\frac{1}{2} \times (\frac{2}{3} \times \frac{3}{4}) = \frac{1}{2} \times (\frac{6}{12}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Both sides are equal.

Example 2:

Let $a = -\frac{1}{3}, b = \frac{3}{5}, c = \frac{5}{2}$

$\left(-\frac{1}{3} \times \frac{3}{5}\right) \times \frac{5}{2} = \left(-\frac{3}{15}\right) \times \frac{5}{2} = \left(-\frac{1}{5}\right) \times \frac{5}{2} = -\frac{5}{10} = -\frac{1}{2}$

$-\frac{1}{3} \times \left(\frac{3}{5} \times \frac{5}{2}\right) = -\frac{1}{3} \times \left(\frac{15}{10}\right) = -\frac{15}{30} = -\frac{1}{2}$

Associative property holds.

Multiplicative Identity

Definition:

When any rational number is multiplied by 1, the result is the number itself.

That means:

a × 1 = a

Example 1:

$(\frac{7}{8}) \times 1 = \frac{7}{8}$

Same number

Example 2:

$(-\frac{5}{6}) \times 1 = -\frac{5}{6}$ Same number

Multiplicative Inverse (Reciprocal)

Definition:

The multiplicative inverse of a rational number $\frac{a}{b}$ is $\frac{b}{a}$, and

$\frac{a}{b} \times \frac{b}{a} = 1$ (as long as a ≠ 0)

That means:

A number multiplied by its reciprocal gives 1.

Example 1:

$(\frac{3}{4}) \times (\frac{4}{3}) = \frac{12}{12} = 1$

Multiplicative inverse works.

Example 2:

$(-\frac{7}{5}) \times (-\frac{7}{5}) = \frac{35}{35} = 1$

Multiplicative inverse holds true.

Summary Table:

Property	Rule	Example
Closure	a × b is a rational number	$(-\frac{2}{3}) \times (\frac{5}{4}) = -\frac{10}{12}$
Commutative	a × b = b × a	$(\frac{1}{2}) \times (\frac{3}{5}) = (\frac{3}{5}) \times (\frac{1}{2})$
Associative	(a × b) × c = a × (b × c)	$(\frac{1}{3} \times \frac{3}{4}) \times 2 = \frac{1}{2}$
Multiplicative Identity	a × 1 = a	$(-\frac{6}{7}) \times 1 = -\frac{6}{7}$
Multiplicative Inverse	$a \times (\frac{1}{a}) = 1$	$(\frac{2}{5}) \times (\frac{5}{2}) = 1$