



## Properties of Multiplication of Rational Numbers

### Closure Property

#### Definition:

If we multiply any two rational numbers, the result is always a rational number.

#### That means:

If  $a$  and  $b$  are rational numbers, then  $a \times b$  is also a rational number.

#### Example 1:

$$\left(\frac{2}{5}\right) \times \left(\frac{3}{4}\right) = \frac{6}{20} = \frac{3}{10} \rightarrow \text{a rational number}$$

So, closure property holds.

#### Example 2:

$$\left(-\frac{7}{8}\right) \times \left(\frac{2}{3}\right) = -\frac{14}{24} = -\frac{7}{12} \rightarrow \text{a rational number}$$

Closure property is true.

### Commutative Property

#### Definition:

Changing the order of numbers does not change the result in multiplication.

#### That means:

$$a \times b = b \times a$$

#### Example 1:

$$\left(\frac{2}{3}\right) \times \left(\frac{5}{7}\right) = \frac{10}{21}$$

$$\left(\frac{5}{7}\right) \times \left(\frac{2}{3}\right) = \frac{10}{21}$$

Both are equal.

#### Example 2:

$$\left(-\frac{4}{9}\right) \times \left(\frac{1}{2}\right) = -\frac{4}{18} = -\frac{2}{9}$$

$$\left(\frac{1}{2}\right) \times \left(-\frac{4}{9}\right) = -\frac{4}{18} = -\frac{2}{9}$$

Commutative property holds.



## Associative Property

### Definition:

When multiplying three or more rational numbers, the way they are grouped does not change the result.

### That means:

$$(a \times b) \times c = a \times (b \times c)$$

### Example 1:

$$\text{Let } a = \frac{1}{2}, b = \frac{2}{3}, c = \frac{3}{4}$$

$$\left(\frac{1}{2} \times \frac{2}{3}\right) \times \frac{3}{4} = \left(\frac{1}{3}\right) \times \frac{3}{4} = \frac{3}{12} = \frac{1}{4}$$

$$\frac{1}{2} \times \left(\frac{2}{3} \times \frac{3}{4}\right) = \frac{1}{2} \times \left(\frac{6}{12}\right) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Both sides are equal.

### Example 2:

$$\text{Let } a = -\frac{1}{3}, b = \frac{3}{5}, c = \frac{5}{2}$$

$$\left(-\frac{1}{3} \times \frac{3}{5}\right) \times \frac{5}{2} = \left(-\frac{3}{15}\right) \times \frac{5}{2} = \left(-\frac{1}{5}\right) \times \frac{5}{2} = -\frac{5}{10} = -\frac{1}{2}$$

$$-\frac{1}{3} \times \left(\frac{3}{5} \times \frac{5}{2}\right) = -\frac{1}{3} \times \left(\frac{15}{10}\right) = -\frac{15}{30} = -\frac{1}{2}$$

Associative property holds.

## Multiplicative Identity

### Definition:

When any rational number is multiplied by 1, the result is the number itself.

### That means:

$$a \times 1 = a$$

### Example 1:

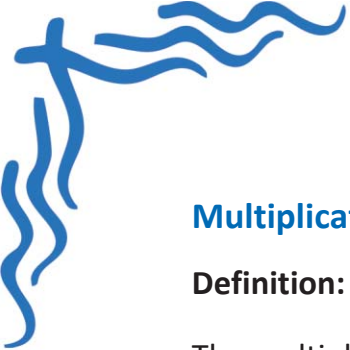
$$\left(\frac{7}{8}\right) \times 1 = \frac{7}{8}$$

Same number

### Example 2:

$$\left(-\frac{5}{6}\right) \times 1 = -\frac{5}{6}$$

Same number



## Multiplicative Inverse (Reciprocal)

### Definition:

The multiplicative inverse of a rational number  $\frac{a}{b}$  is  $\frac{b}{a}$ , and

$$\frac{a}{b} \times \frac{b}{a} = 1 \text{ (as long as } a \neq 0\text{)}$$

### That means:

A number multiplied by its reciprocal gives 1.

### Example 1:

$$\left(\frac{3}{4}\right) \times \left(\frac{4}{3}\right) = \frac{12}{12} = 1$$

Multiplicative inverse works.

### Example 2:

$$\left(-\frac{7}{5}\right) \times \left(-\frac{5}{7}\right) = \frac{35}{35} = 1$$

Multiplicative inverse holds true.

### Summary Table:

Property	Rule	Example
Closure	$a \times b$ is a rational number	$\left(-\frac{2}{3}\right) \times \left(\frac{5}{4}\right) = -\frac{10}{12}$
Commutative	$a \times b = b \times a$	$\left(\frac{1}{2}\right) \times \left(\frac{3}{5}\right) = \left(\frac{3}{5}\right) \times \left(\frac{1}{2}\right)$
Associative	$(a \times b) \times c = a \times (b \times c)$	$\left(\frac{1}{3} \times \frac{3}{4}\right) \times 2 = \frac{1}{2}$
Multiplicative Identity	$a \times 1 = a$	$\left(-\frac{6}{7}\right) \times 1 = -\frac{6}{7}$
Multiplicative Inverse	$a \times \left(\frac{1}{a}\right) = 1$	$\left(\frac{2}{5}\right) \times \left(\frac{5}{2}\right) = 1$