Highest Common Factor (HCF)

The Highest Common Factor (HCF) of two or more numbers is the largest number that divides all the given numbers exactly, without leaving any remainder. In other words, the HCF is the greatest number that is a factor of all the numbers involved.

For example, the HCF of 12 and 18 is 6 because 6 is the greatest number that divides both 12 and 18 exactly.

How to Find the HCF?

There are two main methods for finding the HCF of two or more numbers:

Listing Factors: Write down the factors of each number, then find the largest common factor.

Prime Factorisation: Find the prime factorisation of each number, then identify the common prime factors and multiply them to get the HCF.

Properties of HCF

- The HCF of two or more numbers is always a factor of all the numbers involved.
- The HCF is always less than or equal to the smallest number.
- The HCF of two numbers can never be larger than the smallest of the two numbers.
- If the HCF of two numbers is 1, the numbers are co-prime (they have no common factors other than 1).

Example 1

Question: Find the HCF of 24 and 36 using the listing method.

Solution:

Step 1: List the factors of 24:

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24.

Step 2: List the factors of 36:

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36.

Step 3: Identify the common factors:

Common factors: 1, 2, 3, 4, 6, 12.

Step 4: The highest common factor is 12.

Answer: The HCF of 24 and 36 is 12.

Example 2

Question: Find the HCF of 15 and 25 using the prime factorisation method.

Solution:

Step 1: Find the prime factorisation of 15:

$15 = 3 \times 5$.

Step 2: Find the prime factorisation of 25:

$25 = 5 \times 5$.

Step 3: Identify the common prime factor:

The only common prime factor is 5.

Step 4: The HCF is 5.

Answer: The HCF of 15 and 25 is 5.

Summary Points

- The HCF is the largest number that divides two or more numbers exactly.
- It can be found using the listing method or prime factorisation.
- The HCF is always a factor of the numbers involved and is less than or equal to the smallest number.
- If the HCF is 1, the numbers are called co-prime.