



Some Problems Involving Fractions

i. Definition and Explanation

What are problems involving fractions? These are real-world scenarios or word problems where you need to use your knowledge of fractions to find a solution. Instead of just calculating $\frac{1}{2} + \frac{1}{4}$, you might be asked: "If you ran half a kilometer and then walked another quarter of a kilometer, what is the total distance you covered?"

Solving these problems requires you to:

- Read and Understand the problem to figure out what is being asked.
- Identify which mathematical operation (addition, subtraction, multiplication, or division) is needed.
- Perform the fraction calculation correctly.
- Write the answer in the context of the problem, including units (e.g., kg, meters, hours).

ii. Key Points and Important Terms

Fraction: A number that represents a part of a whole. It has a Numerator (the top number, shows how many parts you have) and a Denominator (the bottom number, shows how many parts the whole is divided into).

Mixed Number: A whole number and a proper fraction combined (e.g., $3\frac{1}{2}$).

Improper Fraction: A fraction where the numerator is greater than or equal to the denominator (e.g., $\frac{7}{2}$).

- **Key Skill:** You must be able to convert between mixed numbers and improper fractions to solve problems.

Lowest Common Denominator (LCD): The smallest number that is a multiple of the denominators of two or more fractions. Essential for adding and subtracting fractions.

Reciprocal: The "flipped" version of a fraction. To find the reciprocal, you swap the numerator and the denominator (e.g., the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$). Essential for dividing fractions.

The word "of": In fraction word problems, "of" almost always means multiplication.

- **Example:** "Find 1441 of 20" means "Calculate 14×20 41 $\times 20$ ".

Simplest Form: Always reduce your final fraction answer to its simplest form (e.g., write 2442 as 1221).

iii. Detailed Examples with Solutions

Example 1: Addition and Subtraction (Combining and Finding the Remainder)

Problem: A painter uses 3883 of a can of paint on a wall and 1441 of the same can on a door.

- What fraction of the paint can did he use in total?
- What fraction of the paint is left?

Solution: a) To find the total, we add the fractions.

Operation: Addition.

Fractions: $38 + 1483 + 41$

Find the LCD: The LCD of 8 and 4 is 8.

Convert fractions: 1441 is equivalent to $1 \times 24 \times 2 = 284 \times 21 \times 2 = 82$.

Add: $38 + 28 = 3 + 28 = 5883 + 82 = 83 + 2 = 85$

Answer: The painter used 5885 of the can of paint.

b) To find what is left, we subtract the used amount from the whole can (1).

Operation: Subtraction.

Calculation: $1 - 581 - 85$

Convert the whole number: 11 is the same as 8888.

Subtract: $88 - 58 = 8 - 58 = 3888 - 85 = 88 - 5 = 83$

Answer: There is 3883 of the paint left in the can.

Example 2: Multiplication (Finding a Part of a Quantity)

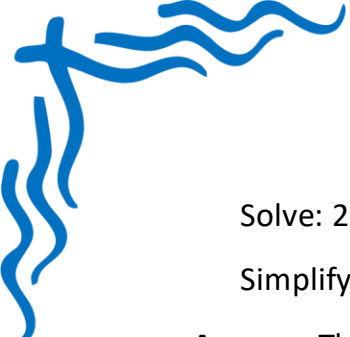
Problem: A school has 600 students. 2552 of the students are in the school band. How many students are in the band?

Solution:

Identify the key phrase: "2552 of the students". This means we need to multiply.

Operation: Multiplication.

Calculation: $25 \times 60052 \times 600$



Solve: $25 \times 6001 = 2 \times 6005 \times 1 = 1200552 \times 1600 = 5 \times 12 \times 600 = 51200$

Simplify: $1200 \div 5 = 240$ $1200 \div 5 = 240$

Answer: There are 240 students in the school band.

Example 3: Division (Finding How Many Fit)

Problem: A chef has a 10-liter container of soup. If each bowl holds $\frac{1}{21}$ of a liter, how many bowls of soup can the chef serve?

Solution:

Understand the question: We are dividing the total amount of soup (10 liters) into smaller, equal portions ($\frac{1}{21}$ liter).

Operation: Division.

Calculation: $10 \div \frac{1}{21} = 210$

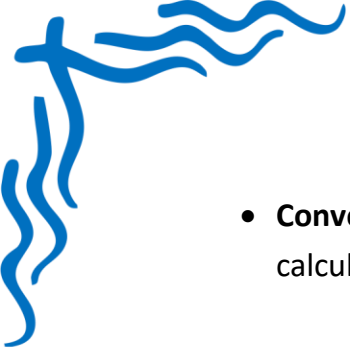
Use the reciprocal: To divide by a fraction, we multiply by its reciprocal. The reciprocal of $\frac{1}{21}$ is 21.

Solve: $10 \times 21 = 210$

Answer: The chef can serve 20 bowls of soup.

iv. Summary of Main Concepts

- **Read Carefully:** The most important step is to understand what the word problem is asking.
- Identify the Operation:
 - **Addition:** For finding a total or combining amounts.
 - **Subtraction:** For finding what's left, the difference, or how much more is needed.
 - **Multiplication:** Usually indicated by the word "of" (e.g., " $\frac{1}{21}$ of 30").
 - **Division:** For splitting a quantity into equal parts (e.g., "How many $\frac{1}{21}$ -cup servings are in 2 cups?").
- **Follow the Rules:**
 - For $+$ / $-$: Find the LCD.
 - For \times : Multiply numerators, multiply denominators.
 - For \div : Keep, Change, Flip (multiply by the reciprocal).



- **Convert and Simplify:** Convert mixed numbers to improper fractions to make calculations easier. Always present your final answer in its simplest form.