Division of a Fraction by Another Fraction

i. Definition and Explanation

What does it mean to divide by a fraction? Conceptually, dividing a fraction by another fraction is asking: "How many times does the second fraction fit into the first fraction?"

- For example, the problem $\frac{1}{2} \div \frac{1}{4}$ is asking, "How many quarters ($\frac{1}{4}$) can fit into a half ($\frac{1}{2}$)?"

The Rule: "Keep, Change, Flip"

The simplest and most reliable way to divide fractions is to turn the division problem into a multiplication problem. We do this using a method often called "Keep, Change, Flip".

- KEEP the first fraction exactly as it is.
- CHANGE the division sign (÷) to a multiplication sign (×).
- FLIP the second fraction upside down. This "flipped" fraction is called the reciprocal.

So, the rule is: To divide by a fraction, multiply by its reciprocal.

ii. Key Points and Important Terms

Fraction: A number representing a part of a whole, written as a numerator over a denominator (e.g., $\frac{a}{b}$).

Dividend: The number being divided. In $\frac{a}{b} \div \frac{c}{d}$, the dividend is $\frac{a}{b}$.

Divisor: The number you are dividing by. In $\frac{a}{b} \div \frac{c}{d}$, the divisor is $\frac{c}{d}$.

Quotient: The result of a division problem.

Reciprocal (or Multiplicative Inverse): A fraction flipped upside down. To find the reciprocal of a fraction, you swap the numerator and the denominator.

- The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.
- The reciprocal of $\frac{1}{8}$ is $\frac{8}{1}$ (or just 8).
- The reciprocal of 5 (which is $\frac{5}{1}$) is $\frac{1}{5}$.

Improper Fraction: A fraction where the numerator is greater than or equal to the denominator (e.g., $\frac{7}{4}$).

Mixed Number: A number consisting of a whole number and a proper fraction (e.g., $1\frac{3}{4}$). You must convert mixed numbers to improper fractions before dividing.

iii. Detailed Examples with Solutions

Example 1: A simple division

Problem: $\frac{1}{2} \div \frac{1}{4}$

KEEP the first fraction.

$\frac{1}{2}$

CHANGE the division sign to multiplication.

$\frac{1}{2} \times$

FLIP the second fraction (find the reciprocal). The reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$.

$\frac{1}{2} \times \frac{4}{1}$

Multiply the numerators and the denominators.

$\frac{(1 \times 4)}{(2 \times 1)} = \frac{4}{2}$

Simplify the result.

$\frac{4}{2} = 2$

Solution: $\frac{1}{2} \div \frac{1}{4} = 2$. (This means two $\frac{1}{4}$ s fit into $\frac{1}{2}$).

Example 2: Division requiring simplification

Problem: $\frac{2}{3} \div \frac{4}{5}$

KEEP $\frac{2}{3}$.

CHANGE ÷ to ×.

FLIP $\frac{4}{5}$ to get $\frac{5}{4}$.

The problem is now: $\frac{2}{3} \times \frac{5}{4}$

Step 4: Multiply.

$\frac{(2 \times 5)}{(3 \times 4)} = \frac{10}{12}$

Step 5: Simplify. Both 10 and 12 are divisible by 2.

$10 \div 2 = 5$

$12 \div 2 = 6$

The simplified fraction is $\frac{5}{6}$.

Solution: $\frac{2}{3} \div \frac{4}{5} = \frac{5}{6}$.

Example 3: Dividing with a mixed number

Problem: $2\frac{1}{3} \div \frac{1}{6}$

Step 1: Convert the mixed number to an improper fraction.

$2\frac{1}{3} = \frac{(2 \times 3 + 1)}{3} = \frac{7}{3}$

The problem is now: $\frac{7}{3} \div \frac{1}{6}$

Step 2: KEEP, CHANGE, FLIP.

KEEP $\frac{7}{3}$.

CHANGE ÷ to ×.

FLIP $\frac{1}{6}$ to get $\frac{6}{1}$.

The problem is now: $\frac{7}{3} \times \frac{6}{1}$

Step 3: Multiply.

$\frac{(7 \times 6)}{(3 \times 1)} = \frac{42}{3}$

Step 4: Simplify.

$42 \div 3 = 14$

Solution: $2\frac{1}{3} \div \frac{1}{6} = 14$.

iv. Summary of Main Concepts

- Dividing by a fraction is the same as multiplying by its reciprocal.
- The easiest way to remember the procedure is "Keep, Change, Flip".
  - KEEP the first fraction.
  - CHANGE the division sign to multiplication.
  - FLIP the second fraction.
- Always convert mixed numbers to improper fractions before you begin.
- After setting up the multiplication, multiply the numerators together and the denominators together.
- Always simplify your final answer to its lowest terms.