Division of Fractions

i. Definition and Explanation

What does it mean to divide by a fraction? Division asks, "How many times does one number fit into another?" When we divide by a fraction, we are asking the same question.

- For example, the problem $6 \div 2$ asks, "How many groups of 2 can you make from 6?" The answer is 3.
- Similarly, the problem $\frac{1}{2} \div \frac{1}{4}$ asks, "How many quarters ($\frac{1}{4}$) can you fit into a half ($\frac{1}{2}$)?" The answer is 2.

The Core Idea: Dividing by a fraction is the same as multiplying by its reciprocal. This is the fundamental rule you will use to solve these problems.

ii. Key Points and Important Terms

Dividend: The number being divided. In $a \div b$, a is the dividend.

Divisor: The number you are dividing by. In $a \div b$, b is the divisor.

Quotient: The result of a division problem.

Reciprocal (or Multiplicative Inverse): A fraction flipped upside down. To find the reciprocal of a fraction, you switch its numerator and denominator.

- The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.
- The reciprocal of $\frac{1}{8}$ is $\frac{8}{1}$.
- The reciprocal of a whole number like 5 (which is $\frac{5}{1}$) is $\frac{1}{5}$.
- Key Property: A number multiplied by its reciprocal always equals 1. (e.g., $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$)

Mixed Number: A number consisting of a whole number and a proper fraction (e.g., 3 ½).

Improper Fraction: A fraction where the numerator is greater than or equal to the denominator (e.g., $\frac{7}{2}$).

iii. Detailed Examples with Solutions

Example 1: Fraction divided by a Fraction

Problem: $\frac{5}{6} \div \frac{1}{3}$

Solution:

KEEP $\frac{5}{6}$.

CHANGE ÷ to ×.

FLIP $\frac{1}{3}$ to $\frac{3}{1}$.

The problem becomes: $\frac{5}{6} \times \frac{3}{1}$

Multiply: $\frac{(5 \times 3)}{(6 \times 1)} = \frac{15}{6}$

Simplify: Both 15 and 6 are divisible by 3. 15 ÷ 3 = 5; 6 ÷ 3 = 2.

Answer: $\frac{5}{2}$ or 2 ½

Example 2: Fraction divided by a Whole Number

Problem: $\frac{3}{4} \div 2$

Solution:

First, write the whole number as a fraction: $2 = \frac{2}{1}$.

The problem is now: $\frac{3}{4} \div \frac{2}{1}$

KEEP $\frac{3}{4}$. CHANGE ÷ to ×. FLIP $\frac{2}{1}$ to $\frac{1}{2}$.

The problem becomes: $\frac{3}{4} \times \frac{1}{2}$

Multiply: $\frac{(3 \times 1)}{(4 \times 2)} = \frac{3}{8}$

Answer: $\frac{3}{8}$

Example 3: Division involving a Mixed Number

Problem: 2 ½ ÷ $\frac{3}{8}$

Solution:

First, convert all mixed numbers to improper fractions.

2 ½ = $\frac{(2 \times 2 + 1)}{2} = \frac{5}{2}$

The problem is now: $\frac{5}{2} \div \frac{3}{8}$

Apply Keep, Change, Flip.

KEEP $\frac{5}{2}$. CHANGE ÷ to ×. FLIP $\frac{3}{8}$ to $\frac{8}{3}$.

The problem becomes: $\frac{5}{2} \times \frac{8}{3}$

Multiply: $\frac{(5 \times 8)}{(2 \times 3)} = \frac{40}{6}$

Simplify: Both 40 and 6 are divisible by 2. 40 ÷ 2 = 20; 6 ÷ 2 = 3.

Answer: $\frac{20}{3}$ or 6 ⅔

iv. Summary of Main Concepts

- **Division as Multiplication:** Dividing by a fraction is the same as multiplying by its reciprocal.
- **Reciprocal:** To find the reciprocal of a fraction, flip the numerator and the denominator.
- **The Method:** Use Keep, Change, Flip.
  1. KEEP the first fraction.
  2. CHANGE the division sign to multiplication.
  3. FLIP the second fraction (the divisor).
- **Mixed Numbers:** Always convert mixed numbers to improper fractions before you do anything else.
- **Simplify:** Always simplify your final answer to its lowest terms.