Product Relationship (Greater/Less Than)

i. Definition and Explanation

What is a Product Relationship? When we multiply a number by another number (a multiplier), the result, called the product, has a predictable relationship with the original number. This relationship tells us if the product will be greater than (>), less than (<), or equal to (=) the original number.

Understanding this concept helps you estimate the result of a multiplication problem without even doing the full calculation!

- **The Core Question**: If we have the problem A × B = C, how does C (the product) compare to A (the original number)?

The answer depends entirely on the value of B (the multiplier).

ii. Key Points and Important Terms

Product: The result of a multiplication problem. (e.g., in 5 × 3 = 15, 15 is the product).

Factors: The numbers that are multiplied together. (e.g., in 5 × 3 = 15, 5 and 3 are the factors).

Multiplier: The factor that we are focusing on to see how it changes the other number.

Inequality: A mathematical statement that compares two values that are not equal.

- > means "Greater Than"
- < means "Less Than"
- = means "Equal To"

iii. Detailed Examples with Solutions (The Rules of Product Relationships)

Let's explore the different cases based on the value of the multiplier.

Case 1: Multiplying by a Number GREATER THAN 1

Rule: The product will be greater than the original number.

Explanation: You are taking the original number and adding it to itself multiple times, making it larger.

Examples:

$8 \times 3 = 24$. Here, $24 > 8$.

$12 \times 1.5 = 18$. Here, $18 > 12$.

$(-5) \times 2 = -10$. Here, $-10 < -5$. Be careful with negatives! The product becomes more negative, which means it is a smaller value.

Case 2: Multiplying by EXACTLY 1

Rule: The product will be equal to the original number.

Explanation: This is the Identity Property of Multiplication. Multiplying by 1 does not change the value.

Examples:

$15 \times 1 = 15$. Here, $15 = 15$.

$-7 \times 1 = -7$. Here, $-7 = -7$.

Case 3: Multiplying by a POSITIVE Number BETWEEN 0 and 1 (Proper Fractions or Decimals)

Rule: The product will be less than the original number.

Explanation: You are finding a "part" or a "fraction" of the original number, which makes it smaller.

Examples:

$20 \times 0.5 = 10$. Here, $10 < 20$. (Multiplying by 0.5 is the same as taking half).

$16 \times (\frac{1}{4}) = 4$. Here, $4 < 16$. (Multiplying by $\frac{1}{4}$ is the same as dividing by 4).

$(-10) \times 0.5 = -5$. Here, $-5 > -10$. Be careful with negatives! The product becomes less negative, which means it is a greater value.

Case 4: Multiplying by EXACTLY 0

Rule: The product will always be 0.

Explanation: This is the Zero Property of Multiplication. Any number multiplied by zero is zero.

Example:

$1{,}250 \times 0 = 0$. Here, $0 < 1{,}250$.

Case 5: Multiplying a POSITIVE Number by a NEGATIVE Number

Rule: The product will always be negative, and therefore less than the original positive number.

Explanation: A positive times a negative is always a negative. Any negative number is less than any positive number.

Example:

10 × (-4) = -40. Here, -40 < 10.

iv. Summary of Main Concepts

Use this table as a quick guide to product relationships. We are comparing the Product to the Original Number.

If you multiply an Original Number by...	The Product will be...	Example (Original Number = 10)	Example (Original Number = -10)
A number > 1	Greater than the original.	10 × 2 = 20 (20 > 10)	(-10) × 2 = -20 (-20 < -10) (Less than!)
Exactly 1	Equal to the original.	10 × 1 = 10 (10 = 10)	(-10) × 1 = -10 (-10 = -10)
A positive number between 0 and 1	Less than the original.	10 × 0.5 = 5 (5 < 10)	(-10) × 0.5 = -5 (-5 > -10) (Greater than!)
Exactly 0	0 (which is less than a positive original, greater than a negative original)	10 × 0 = 0 (0 < 10)	(-10) × 0 = 0 (0 > -10)
A negative number	Less than the original (if original is positive).	10 × (-2) = -20 (-20 < 10)	Depends on the negative multiplier

Key Takeaway: The value of the multiplier is the key. Be especially careful when working with negative numbers, as the rules for "greater than" and "less than" can feel reversed. Always think about the number line.