Multiplication of Fractions

i. Definition and Explanation

What is Multiplication of Fractions? Multiplying fractions means finding a part of a part. When you multiply a fraction by another fraction, you are finding a fractional part of that fraction.

The word "of" in mathematics often means multiplication.

- For example, finding $\frac{1}{2}$ of $\frac{1}{2}$ of a pizza is the same as calculating $\frac{1}{2} \times \frac{1}{2}$. The answer is $\frac{1}{4}$, which is a smaller piece of the whole pizza.

Unlike addition and subtraction, you do not need a common denominator to multiply fractions.

ii. Key Points and Important Terms

Numerator: The top number in a fraction. It tells you how many parts you have.

Denominator: The bottom number in a fraction. It tells you how many equal parts the whole is divided into.

Proper Fraction: A fraction where the numerator is smaller than the denominator (e.g., $\frac{3}{5}$). Its value is less than 1.

Improper Fraction: A fraction where the numerator is greater than or equal to the denominator (e.g., $\frac{7}{4}$). Its value is 1 or greater.

Mixed Number: A number consisting of a whole number and a proper fraction (e.g., $2\frac{1}{3}$).

Simplify / Reduce: To write a fraction in its simplest form by dividing both the numerator and the denominator by their greatest common factor (GCF).

Reciprocal: The "flipped" version of a fraction. The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$. (This is more important for division, but good to know).

iii. The Process: How to Multiply Fractions

Multiplying fractions is a straightforward, three-step process.

Step 1: Multiply the Numerators Take the top numbers of the fractions and multiply them together. This will be the numerator of your answer.

Step 2: Multiply the Denominators Take the bottom numbers of the fractions and multiply them together. This will be the denominator of your answer.

Step 3: Simplify the Result If the resulting fraction can be simplified, reduce it to its lowest terms.

Formula: $\frac{a}{b} \times \frac{c}{d} = \frac{(a \times c)}{(b \times d)}$

iv. Detailed Examples with Solutions

Example 1: Multiplying two proper fractions

Calculate: $\frac{2}{3} \times \frac{4}{5}$

Multiply the numerators: 2 × 4 = 8

Multiply the denominators: 3 × 5 = 15

Write the new fraction: $\frac{8}{15}$. Can this be simplified? No, 8 and 15 have no common factors other than 1.

Solution: $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

Example 2: Multiplying a fraction by a whole number

Calculate: $\frac{3}{4} \times 7$

First, turn the whole number into a fraction by putting it over 1: $7 = \frac{7}{1}$

Now the problem is: $\frac{3}{4} \times \frac{7}{1}$

Multiply the numerators: 3 × 7 = 21

Multiply the denominators: 4 × 1 = 4

Write the new fraction: $\frac{21}{4}$. This is an improper fraction. You can leave it as is or convert it to a mixed number.

To convert $\frac{21}{4}$ to a mixed number, ask: "How many times does 4 go into 21?" It goes in 5 times (4 × 5 = 20) with a remainder of 1. So, the mixed number is $5\ \frac{1}{4}$.

Solution: $\frac{3}{4} \times 7 = \frac{21}{4}$ or $5\ \frac{1}{4}$

Example 3: Multiplying two mixed numbers

Calculate: $1\ \frac{1}{2} \times 2\ \frac{2}{3}$

First, convert both mixed numbers into improper fractions.

- $1\frac{1}{2}$ becomes $\frac{(1\times 2+1)}{2}=\frac{3}{2}$
- $2\frac{2}{3}$ becomes $\frac{(2\times 3+2)}{3}=\frac{8}{3}$

Now the problem is: $\frac{3}{2}\times\frac{8}{3}$

Multiply numerators: $3 \times 8 = 24$

Multiply denominators: $2 \times 3 = 6$

Write the new fraction: $\frac{24}{6}$. Simplify this fraction. $24 \div 6 = 4$.

Solution: $1\frac{1}{2}\times 2\frac{2}{3}=4$

v. Summary of Main Concepts

- **The Golden Rule:** To multiply fractions, multiply the numerators, then multiply the denominators, and finally simplify. $\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$.
- **"Of" means Multiply:** When you see the word "of" in a word problem with fractions, it's a clue to multiply.
- **Whole Numbers:** Treat a whole number as a fraction over 1 (e.g., $5 = \frac{5}{1}$).
- **Mixed Numbers:** MUST be converted to improper fractions before multiplying.
- **Simplifying:** You can simplify the final answer, or you can simplify before multiplying by cross-cancelling. Cross-cancelling makes the numbers smaller and easier to work with.
- **The Answer's Size:** Multiplying by a number greater than 1 makes the result bigger. Multiplying by a number less than 1 (a proper fraction) makes the result smaller.