Even and Odd Numbers

i. Definition and Explanation

At its core, the concept of even and odd numbers is about whether an integer can be perfectly divided by 2.

Even Numbers:- An even number is an integer that is exactly divisible by 2, meaning it can be divided by 2 with no remainder.

- **Key Idea:** If you have an even number of items, you can always group them into pairs with none left over.
- **The Last Digit Rule:** A number is even if its last digit is 0, 2, 4, 6, or 8.
- **Algebraic Definition:** An integer n is even if it can be written in the form $n = 2k$, where k is any integer.

Examples: 8, 14, 76, 150, -22, -100.

Odd Numbers:- An odd number is an integer that is not exactly divisible by 2, meaning there is a remainder of 1 when it is divided by 2.

- **Key Idea:** If you have an odd number of items, you will always have one left over after making as many pairs as possible.
- **The Last Digit Rule:** A number is odd if its last digit is 1, 3, 5, 7, or 9.
- **Algebraic Definition:** An integer n is odd if it can be written in the form $n = 2k + 1$, where k is any integer.

Examples: 7, 15, 83, 299, -3, -101.

ii. Key Points and Important Terms

Integer: A whole number (not a fraction or decimal) that can be positive, negative, or zero. The concept of even and odd applies only to integers.

Divisible: A number is divisible by another number if the division results in an integer with a remainder of 0.

Remainder: The amount "left over" after a division operation. For even/odd numbers, we only care if the remainder is 0 or 1 when dividing by 2.

Zero (0) is an Even Number: This is a very important point. Zero is even because $0 \div 2 = 0$ with a remainder of 0. It also fits the algebraic definition: $0 = 2 \times 0$.

Negative Numbers: Negative integers can also be even or odd. The same rules apply. For example, -4 is even ($-4 = 2 \times -2$) and -7 is odd ($-7 = 2 \times -4 + 1$).

Properties of Operations with Even and Odd Numbers

Understanding these rules can help you solve problems without doing the full calculation.

Operation	Numbers	Result	Example
Addition	Even + Even	Even	4 + 6 = 10
	Odd + Odd	Even	3 + 5 = 8
	Even + Odd	Odd	4 + 3 = 7
Subtraction	Even - Even	Even	8 - 2 = 6
	Odd - Odd	Even	9 - 3 = 6
	Even - Odd	Odd	10 - 3 = 7
	Odd - Even	Odd	9 - 2 = 7
Multiplication	Even × Even	Even	2 × 4 = 8
	Odd × Odd	Odd	3 × 5 = 15
	Even × Odd	Even	4 × 3 = 12

Key Multiplication Takeaway: If you multiply any integer by an even number, the result will always be even.

iii. Detailed Examples with Solutions

Example 1: Identifying Numbers Is the number 3,456 even or odd?

Solution:

Look at the last digit. The last digit is 6. Since 6 is an even digit, the number 3,456 is even.

Verification: 3456 ÷ 2 = 1728 (no remainder).

Example 2: Using Properties of Addition Is the sum of 147 and 892 even or odd? (Solve without finding the actual sum).

Solution:

Identify the nature of each number.

147 ends in 7, so it is Odd.

892 ends in 2, so it is Even.

Apply the addition rule: Odd + Even = Odd.

Therefore, the sum of 147 and 892 is odd.

Example 3: Using Properties of Multiplication What is the nature (even or odd) of the product of 51, 23, and 9?

Solution:

Identify the nature of each number.

51 is Odd.

23 is Odd.

9 is Odd.

Apply the multiplication rule:

First, multiply the first two: Odd × Odd = Odd.

Now, multiply that result by the third number: Odd × Odd = Odd.

Therefore, the product is odd.

iv. Summary of Main Concepts

- **Even Numbers:** Integers ending in 0, 2, 4, 6, or 8. They are divisible by 2.
- **Odd Numbers:** Integers ending in 1, 3, 5, 7, or 9. They have a remainder of 1 when divided by 2.
- **Zero:** Is an even number.
- **Scope:** The even/odd classification applies only to integers (positive, negative, and zero).
- **Key Rules:**
  - Odd + Odd = Even
  - Even + Even = Even
  - Odd + Even = Odd
  - Odd × Odd = Odd
  - Even × (Any Integer) = Even