# Properties of Angles with Parallel Lines

i. Definition and Explanation

This topic explores the special relationships that are created when a straight line crosses two parallel lines.

- **Parallel Lines:** Two lines on a plane that are always the same distance apart and will never intersect (never cross), no matter how far they are extended. We use arrows on the lines to show they are parallel.

- **Transversal:** A line that intersects (crosses) two or more other lines at different points.
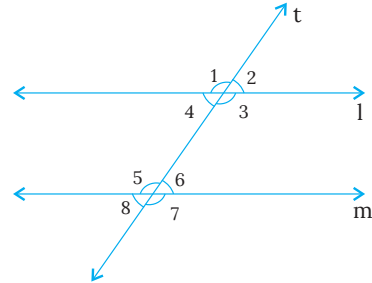
When a transversal intersects two parallel lines, it creates eight angles. These angles have predictable and useful relationships with each other.

ii. Key Points and Important Terms

Using the diagram above, we can define the different types of angles.

Interior Angles: Angles that lie between the two parallel lines.

- **Angles:** $\angle 3, \angle 4, \angle 5, \angle 6$

Exterior Angles: Angles that lie outside the two parallel lines.

- **Angles:** $\angle 1, \angle 2, \angle 7, \angle 8$

Corresponding Angles: Angles that are in the same relative position at each intersection. Think of them as being in the "same corner".

- **Pairs:** ($\angle 1$ and $\angle 5$), ($\angle 2$ and $\angle 6$), ($\angle 3$ and $\angle 7$), ($\angle 4$ and $\angle 8$)
- **Property:** Corresponding angles are equal.

Alternate Interior Angles: Angles that are on opposite sides of the transversal and are between the parallel lines. They form a "Z" shape.

- **Pairs:** ($\angle 3$ and $\angle 6$), ($\angle 4$ and $\angle 5$)
- **Property:** Alternate interior angles are equal.

Alternate Exterior Angles: Angles that are on opposite sides of the transversal and are outside the parallel lines.

- **Pairs:** ($\angle 1$ and $\angle 8$), ($\angle 2$ and $\angle 7$)

- **Property:** Alternate exterior angles are equal.

Consecutive Interior Angles (or Co-interior Angles): Angles that are on the same side of the transversal and are between the parallel lines. They form a "C" or "U" shape.

- **Pairs:** (∠3 and ∠5), (∠4 and ∠6)

- **Property:** Consecutive interior angles are supplementary (they add up to 180°).

Vertically Opposite Angles: Angles opposite each other when two lines cross. They are always equal.

- **Pairs:** (∠1 and ∠4), (∠2 and ∠3), (∠5 and ∠8), (∠6 and ∠7)

Angles on a Straight Line (Linear Pair): Angles that are adjacent to each other on a straight line. They are supplementary (add up to 180°).

- **Examples:** ∠1 + ∠2 = 180°, ∠5 + ∠6 = 180°

## iii. Detailed Examples with Solutions

Example 1: Finding a Corresponding Angle

Problem: In the diagram below, line A is parallel to line B. If ∠x = 125°, what is the measure of ∠y?

Solution:

Identify the relationship: ∠x and ∠y are in the same position at each intersection. They are corresponding angles.

Apply the property: The property of corresponding angles states that they are equal.

Answer: Therefore, ∠y = ∠x = 125°.
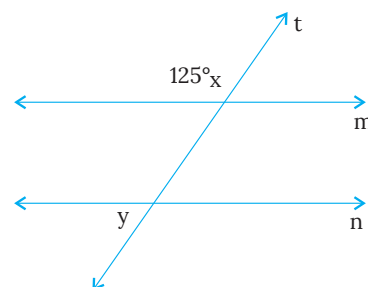
Example 2: Using Two Properties

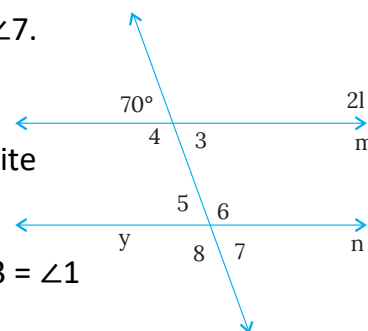**Problem:** In the diagram, l || m. If ∠1 = 70°, find the measure of ∠7.

Solution:

Identify a starting relationship: ∠1 and ∠3 are vertically opposite angles.

Apply the property: Vertically opposite angles are equal, so ∠3 = ∠1 = 70°.

Identify the next relationship: ∠3 and ∠6 are alternate interior angles.

Apply the property: Alternate interior angles are equal, so ∠6 = ∠3 = 70°.

Identify the final relationship: ∠6 and ∠7 are angles on a straight line.

Apply the property: Angles on a straight line are supplementary (add to 180°). So, ∠6 + ∠7 = 180°.

Calculate: 70° + ∠7 = 180°.

Answer: ∠7 = 180° - 70° = 110°.

Example 3: Solving with Algebra

Problem: Lines p and q are parallel. Find the value of 'x'.

```
                /
 p ----------/----------- >
          /  (2x + 30)°
        /
      /
    /
 q -----/------------- >
    /  (3x - 10)°
  /
```
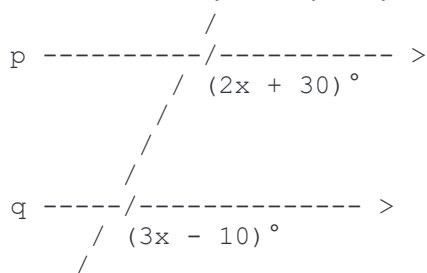
Solution:

Identify the relationship: The two angles are on the same side of the transversal and between the parallel lines. They are consecutive interior angles.

Apply the property: Consecutive interior angles are supplementary (add up to 180°).

Set up the equation: (2x + 30) + (3x - 10) = 180.

Solve the equation:

Combine like terms: 5x + 20 = 180

Subtract 20 from both sides: 5x = 160

Divide by 5: x = 32

Answer: The value of x is 32.

iv. Summary of Main Concepts

If two parallel lines are cut by a transversal, then:

- Corresponding Angles are EQUAL. (F-Angles)

- Alternate Interior Angles are EQUAL. (Z-Angles)

- Alternate Exterior Angles are EQUAL.

- Consecutive Interior Angles are SUPPLEMENTARY (add up to 180°). (C-Angles)