Pick Patterns and Reveal Relationships

i. Definition and Explanation

In mathematics, a pattern is a set of numbers, shapes, or objects that are arranged following a specific, predictable rule. The skill of "Picking Patterns and Revealing Relationships" is about being a math detective. Your job is to:

- Observe a sequence and identify how it changes from one step to the next.
- Describe this change in words.
- Generalize the pattern by creating a mathematical rule or formula (an algebraic expression). This is the "relationship."
- Use this rule to predict future terms in the pattern without having to write them all out.

The "relationship" is the connection between a term's position in the pattern and its actual value.

ii. Key Points and Important Terms

Pattern: A sequence of items that follow a rule.

- **Numerical Pattern:** 2, 4, 6, 8, ...
- **Geometric Pattern:** A sequence of shapes that change in a predictable way.

Term: Each individual item (number or shape) in a pattern.

Term Number (n): The position of a term in the pattern. This is your input. (e.g., 1st term, 2nd term, 3rd term...). We use the variable 'n' to represent the term number.

Term Value (T): The value of the term at a specific position. This is your output. (e.g., in the pattern 2, 4, 6, the value of the 3rd term is 6). We often use the variable 'T' to represent the term value.

Common Difference: In many patterns, this is the constant number you add or subtract to get from one term to the next.

Relationship (or Rule): An algebraic expression or formula that connects the Term Number (n) to the Term Value (T). It allows you to find any term in the pattern.

iii. Detailed Examples with Solutions

Example 1: A Simple Increasing Numerical Pattern

Pattern: 5, 8, 11, 14, ...

Goal: Find the rule for this pattern and determine the 50th term.

Solution:

Create a Table Organize the information in a table to see the relationship clearly.

Term Number (n)	Term Value (T)
1	5
2	8
3	11
4	14

Find the Common Difference Look at how the Term Value (T) changes each time.

- From 5 to 8, we add 3.
- From 8 to 11, we add 3.
- From 11 to 14, we add 3. The common difference is +3.

Start Building the Rule Since the common difference is 3, the rule will involve "3n" (3 times the term number). Let's test this part of the rule.

n	3n	Term Value (T)	How to get from 3n to T?
1	3(1) = 3	5	3 + 2 = 5
2	3(2) = 6	8	6 + 2 = 8
3	3(3) = 9	11	9 + 2 = 11

We see that we always have to add 2.

Write the Final Rule The relationship is: The Term Value is 3 times the Term Number, plus 2. Rule: T = 3n + 2

Use the Rule to Predict Find the 50th term by substituting n = 50 into the rule.

- T = 3(50) + 2
- T = 150 + 2
- T = 152

Answer: The 50^{th} term is 152.

iv. Summary of Main Concepts

- Patterns are all around us and follow predictable rules.
- The goal is to find the relationship between a term's position (n) and its value (T).
- **The Four-Step Process:**
  - **Table:** Organize the pattern in a T-Chart (n vs. T).
  - **Difference:** Find the common difference (how much it changes by each step).
  - **Rule:** Use the common difference to build an algebraic rule (e.g., T = ...n ...). Test it and find the constant that needs to be added or subtracted.
  - **Predict:** Use your final rule to find the value of any term in the sequence.
- An algebraic rule is powerful because it lets you find any term without calculating all the terms before it.