Simplification of Algebraic Expressions

i. Definition and Explanation

What is an Algebraic Expression? An algebraic expression is a mathematical phrase that includes numbers, variables (letters like x or y), and operation signs $(+, -, \times, \div)$. It does not have an equals sign (=), which makes it different from an equation.

• Example: 3x + 7, 5y - 2a + 10, 4(z - 1)

What does it mean to "Simplify" an Expression? Simplifying an algebraic expression means rewriting it in the most compact or efficient way possible, without changing its value. Think of it like tidying up a messy room—you group similar items together to make it neater and easier to understand.

The main way we simplify is by combining like terms.

ii. Key Points and Important Terms

To understand simplification, you must know the parts of an expression:

Term: A single number, a single variable, or a combination of numbers and variables multiplied together. Terms are separated by + or - signs.

• In the expression 5x - 3y + 8, the terms are 5x, -3y, and 8.

Variable: A letter used to represent an unknown number.

• Examples: x, y, a, m.

Coefficient: The number that is multiplied by a variable.

- In 5x, the coefficient is 5.
- In -3y, the coefficient is -3.
- In a, the coefficient is an "invisible" 1.

Constant: A term that is just a number and has no variable. Its value never changes.

• In 5x - 3y + 8, the constant is 8.

Like Terms: This is the most important concept! Like terms are terms that have the exact same variable(s) raised to the exact same power(s). The coefficients can be different.

Examples of Like Terms:

- 7x and 2x (same variable x)
- -5a and a (same variable a)
- 4y² and 9y² (same variable y to the same power 2)
- 8 and -3 (both are constants)

Examples of Unlike Terms:

- 7x and 7y (different variables)
- 5x and 5x² (different powers)
- 3a and 10 (one has a variable, one is a constant)

The Golden Rule of Simplification: You can only add or subtract like terms.

iii. Detailed Examples with Solutions

Example 1: Basic Combination Simplify: 4a + 9a

Solution:

Identify if the terms are "like terms." (Yes, both have the variable a).

Add their coefficients: 4 + 9 = 13.

Keep the variable the same.

Answer: 13a

Example 2: Combining with Subtraction Simplify: 10x - 3x + 2

Solution:

Identify the like terms: 10x and -3x. The term 2 is a constant and has no other like terms.

Combine the like terms by subtracting their coefficients: 10 - 3 = 7.

Keep the variable x.

Bring down the constant term.

Answer: 7x + 2

Example 3: Grouping Multiple Like Terms Simplify: 3y + 8 + 5y - 2

Solution:

Identify and group the like terms. It helps to circle or underline them.

• Variable terms: 3y and +5y

• Constant terms: +8 and -2

Combine the y terms: 3y + 5y = 8y.

Combine the constant terms: 8 - 2 = 6.

Answer: 8y + 6

Example 4: Using the Distributive Property The Distributive Property states that a(b + c) = ab + ac. You "distribute" the term outside the parentheses to every term inside.

Simplify: 5(x + 2)

Solution:

Multiply the 5 by the first term inside, $x: 5 \times x = 5x$.

Multiply the 5 by the second term inside, $+2: 5 \times 2 = 10$.

Combine the results.

Answer: 5x + 10

Example 5: Putting It All Together Simplify: 4(2m - 3) + 3m

Solution:

Step 1: Distribute. Apply the distributive property first.

- $4 \times 2m = 8m$
- $4 \times -3 = -12$
- The expression becomes: 8m 12 + 3m

Step 2: Identify and combine like terms.

- The like terms are 8m and +3m.
- 8m + 3m = 11m

Step 3: Write the final simplified expression.

• Bring down the constant term -12.

Answer: 11m - 12

iv. Summary of Main Concepts

- **Goal:** To make an algebraic expression shorter and simpler by combining terms.
- **Like Terms:** Terms with the exact same variable and exponent. This is the key to simplification.

• The Process:

- **Distribute:** If there are parentheses, multiply the outside term by every term inside.
- **Identify:** Find and group all the like terms. Remember to keep the sign with the term.
- **Combine:** Add or subtract the coefficients of the like terms. The variable part stays the same.
- The Golden Rule: You can ONLY add or subtract like terms. You cannot combine x with y, or x with x².